ESTIMATION OF INTEREST RATE RISK ATTENDING THE BONDS LOAN

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Abstract

The purpose of this paper is to identify and analyze the techniques and methods to estimate the interest rate risk attending a bonds loan, in order to be able to offer a useful and efficient tool for underlying and improve the investment financing decisions. Methodologically, the research has recourse to adequate methods of assessment of financial flows: the binomial method, the decision tree, the Monte Carlo simulation and the specific parameters for updating the cash flow, as well as the statistical measurement of output's volatility through the chronological series method. The research relieves placing the interest rate risk's estimation in decision process of the companies; systematization and clarification of ways to estimate the interest rate risk in relation with the strategy of the company and the investment programs' financing source; the conveyance and estimation of bond's price volatility as a fundamental of the interest rate risk; quantification of sensitivity of held position to interest rate change. The success in interest rate risk management can't be achieved unless a good quantification is made, allowing afterwards diminishing the risk exposure to admissible level.

Key words: interest rate, risk, bond, valuation, duration

INTRODUCTION

The ideal of every manager is to find some interest rate model that explains, but especially forecasts, the behavior of interest rates. Alas, none presently do so [2]. That is not to say that interest rate behavior cannot be explained. Various economical, financial, political or social factors explain some of the behavior of interest rates over history. Interest rates can be seen to fluctuate from day to day, intra-day or longer period, due to the changes in monetary politics.

The implications are clear. Corporations with debts can rely only on uncertainty as far as their cost of funds and payment aschedule (cash flows) are concerned. Investors with portfolios of bonds will also have to manage the constant volatility in interest rates, in order to protect the value of their investments. Either issuer or underwriter, seller or buyer, portfolio owner or manager, the holder of a bond position is eager for controlling interest rate risk, impossible to be done without passing through risk quantification phase.

Interest rate risk assessment that follows bond loan is an essential phase in the ample interest rate risk management process, retraced in order to achieve the optimization of investments fund decisions.

MATERIAL AND METHOD

The comprehension and the knowledge of interest rate risk management start up with the wise attitude on the wide nature of this risk. Interest rate risk could be identify with the possibility that the holder of a debenture and/or debt, a present or a future one, with fixed or flexible interest rate, to record a loss due to the future changes in the market interest rate (decrease, increase or changing in interest rate structure outwards or at term) [5].

For *the bond loans drawn up with fixed interest rate*, the decrease of market interest rate involves an increase of the update value of debts. The increase of the market value of debts leads to the decrease of the common stock value, calculated as difference between assets and liabilities. The effect is similar for debts drawn up with variable interest rate as the matching is done quarterly, half/yearly or yearly, and the loss could be marked inside the period.

As underwriter for bond loans with fixed interest rate, the company will bear an opportunity loss when the market interest rate grows up. The company will collect much less from financial incomes then the credit market could offer in present. The decrease of market value for these assets leads to the same loss of the common stock value, as result of the ineffective management of interest rate. The effects are the same for the investments done with variable interest rates.

The key to measuring the potential loss of a position is how good the estimate is of the value of the position after an adverse rate change. The fundamental relationship for the loss of a position result from the adverse change in interest rate is: *potential loss of a position = value of position after adverse rate change – current market value of position*. [4]

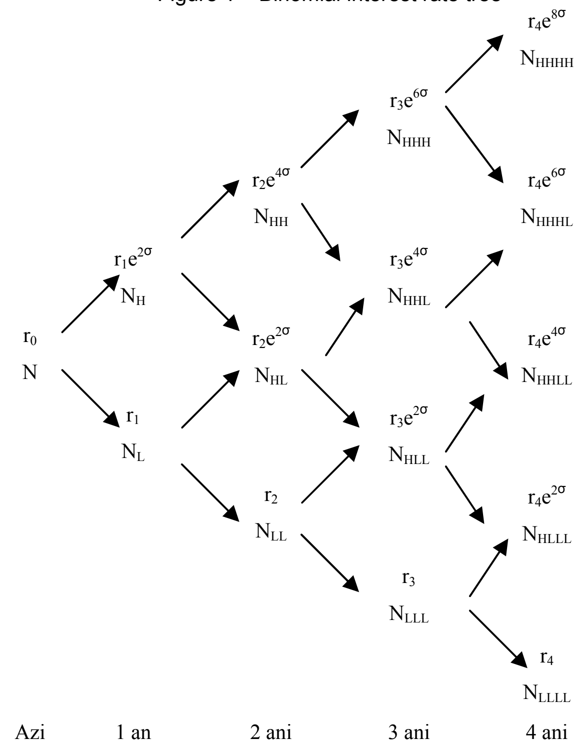
A *bond valuation model* is used to determine the value of a position. For any bond in which *neither the issuer nor the investor can alter the repayment of the principal* before its contractual date, the valuation process is easier. The fundamental principle of valuation is that the value of a financial asset is the present value of the expected cash flow [1]. Cash flow in the case of a bond derives from interest income or repayment of principal. Once the cash flow for a bond is estimated, the next step is to determine the appropriate interest rate to use to discount the cash flow. The traditional practice in valuation has been to discount every cash flow of a bond by the same interest rate (discount rate). The fundamental flaw of the traditional approach is that it views each security as the same package of cash flows. The proper way to view a bond is as a package of zero-coupon instruments. The reason that this is the proper way is because it does not allow a market participant to realize an arbitrage profit. By viewing any financial asset in this way, a consistent valuation framework can be developed.

The starting point for the determination of the appropriate rate is the *theoretical spot rate on default-free securities*. Since Treasury securities are viewed as default-free securities, the theoretical spot rates on these securities are the benchmark rates. They could be obtain from the Treasury Yield Curve, that is the graphical depiction of the relationship between the yield on zero-coupon Treasury securities of different maturities, typically constructed from the on-the-run Treasury issues. For a non-Treasury bond, the theoretical value must reflect not only the spot rate for default-free bonds, but also a *risk premium* to reflect default risk and any options embedded in the issue. There is no reason to expect the credit spread to be the same regardless of when the cash flow is expected to be received, that involve the necessity to build up a term structure for credit spreads.

For a correct estimation of cash flows for *bonds with embedded options*, is necessary to induct in analysis the possibility that interest rates will change in future and the way these changes will affect the issuer of the bond or the bondholder decision to exercise an option. This is done in valuation methodologies by introducing a parameter that reflects the interest rate volatility. There are two main approaches to the valuation of bonds with embedded options: the binomial method and the Monte Carlo simulation method. [4]

The binomial method is a technique for valuing callable and putable bonds. Once we allow for embedded options, consideration must be given to interest rate volatility. This can be done by introducing a *binomial interest rate tree* [2]. This tree is nothing more than a graphical depiction of the interest rates over time based on some assumption about interest rate volatility. How this tree is constructed is illustrated below:

Figure 1 – Binomial interest rate tree



The point denoted by N is the root of the tree and correspond to the current one year spot rate, noted by r_0. What we have assumed in creating this tree is that the one year rate can take on two possible values the next period and the two rates have the same probability of occurring. One rate will be higher than the other. It is assumed that the one year rate can evolve over time based on a random process called a *lognormal random walk* with a certain volatility.

We use the following notation to describe the tree in the first year:

σ = assumed volatility of the one year rate;
$r_{1,L}$ = lower one year rate one year from now;
$r_{1,H}$ = higher one year rate one year from now.

The relationship between $r_{1,L}$ and $r_{1,H}$ is as follows:

$r_{1,H} = r_{1,L}\ (e^{2\sigma})$, where *e* is the base of the natural logarithm 2.71828, so $r_{1,H}$ is replaced by $r_{1,L}\ (e^{2\sigma})$ in the tree.

Each node represents a time period that is equal to one year from the node to its left. Each node is labeled with N, representing node, and a subscript that indicates the path that one year interest rate took to get to that node and is calculated using the same lognormal random walk describe earlier. L represents the lower of the two one-year rates and H represents the higher.

To find the value of the bond at a node, we first calculate the bond's value at the two nodes to the right of the node we are interest in using the following notations: V_H = bond's value for the higher one-year rate; V_L = bond's value for the lower one-year rate; C = coupon payment.

$$\textit{Bond`s value at a node} = \frac{1}{2}\left[\frac{V_H + C}{(1+r_x)} + \frac{V_L + C}{(1+r_x)}\right]$$

The volatility assumption has an important impact on the theoretical value. More specifically, the higher the expected volatility (σ), the higher the value of a bond.

The second method for valuing bonds with embedded options is *Monte Carlo simulation* [1]. The method involves simulating a sufficiently large number of potential interest rate paths in order to assess the value of a security along these different paths. This method is most flexible of the two valuation methodologies for valuing interest rate sensitive instruments where the history of interest rate is important. Mortgage-backed securities are commonly valued using this method, but it is use by some dealers to value callable and putable bonds too. The simulating works by generating a set of cash flows based on future simulated refinancing rates, that involves simulating of anticipated payment rates.

For an effective control of exposure to the interest rate risk, *the quantification of the sensitivity of the held position to the changes in interest rate* is needed [4]. The choice and implementation of the most suitable trading strategy or hedging is determined by finding a mean to measure the volatility of a bond's price. The most frequently used indicators measuring the degree of exposure of these financial tools to the change of yield and, thus, of interest rate are the following: value of a „basic point"; yield variation at the price change; duration and convexity.

1. Value of a „basic point"

The value of a „basic point" [3] represents the absolute (not relative) change of a bond's price when the yield changes by 1 basic point (by 0.01%). The price of a „basic point" is the same at the growth by 0.01% of the yield and at the drop by 0.01% of it. For six bonds taken into account, the price of a basic point can be determined as follows:

Table 1
The value of a basic point

Bond	Initial price at 8% yield	Price at 8.01% yield	Price of a basic point
Coupon 8%, 5 years maturity	100,0000	99,9595	0,0405
Coupon 8%, 20 years maturity	100,0000	99,9019	0,0981
Coupon 5%, 5 years maturity	87,8337	87,7961	0,0376
Coupon 5%, 20 years maturity	70,3108	70,2340	0,0768
Zero-coupon, 5 years maturity	67,5564	67,5239	0,0325
Zero-coupon, 20 years maturity	20,8289	20,7889	0,0400

The indicator can't serve, though, for the calculation of the absolute change of the price when the yield changes by a larger number of basic points (100, for instance).

2. Yield variation at the price change

This indicator assumes the calculation of the yield until the maturity if the bond's price decreases by X units. The difference between the initial yield and the one obtained in that way represents the yield's variation at a change by X units of the bond's price.

In USA, the corporate and municipal bonds are traded in eighths of percentage points of the nominal value, i.e. changes smaller than 1/8 * 1% * 100 USD nominal value = 0.125 USD are not accepted. As a consequence, the investors want to know what yield variation draws in the price's change by 0.125 USD, or which is „the yield of an eighth". The yield variation at a decrease by an eighth of the price of two hypothetical bonds is the following:

Table 2
Example setting the yield variation at the drop by an eighth of the bond's price

Bond	Initial yield (%)	Initial price	Initial price minus an eighth	The new yield	Yield variation
7%/20	5,000	125,10	124,975	5,00872	0,00872
7%-5	5,000	108,75	108,625	5,02760	0,0276

3. *Duration*

The most relevant way for measuring the sensitivity of a bond's price to the changes in the interest rate is the one setting how the price will change, as percentage from the actual price, if the interest rate changes by a smaller number of basic points.

Duration can be defined as the approximate percentage change of the bond's price at the parallel move of the yield's curve by 100 basic points. For instance, a bond with a life of 5 will change its price by approx. 5% at the parallel move of the yield's curve by 100 basic points. At the parallel move by 50 basic points of the yield's curve, the bond's price will change by about 2.5%.

A manager that anticipates a drop of the interest rate will extend (increase) the portfolio's duration. Assuming that the manager increases the portfolio's life from 4 to 6, it means that at a decrease of the interest rate by 100 basic points, the portfolio's value will increase by another 2%, as compared to the case in which the duration would have remain unchanged.

The form preferred by practitioners is *modified duration*, meaning by that the approximate percentage change of the bond's price at the parallel move of the yield's curve by 100 basic points, *assuming that the cash flows of the bond do not change when the curve moves*. The change of the bond's price when the curve is moved by a smaller number of basic points is due only to the update by the new level of the yield.

The assumption that the cash flows will not change when the yield curve is moving is valid only when one talks about bonds without included options, such as, for instance, the Treasury bonds, because the payments made by the state to the bond holders do not change when the yield curve is changed. One cannot say the same neither in the case of bought back and paid back bonds, nor in the case of mortgage bonds. For this type of bonds, a yield change will draw up an alteration of the expected cash flows.

The assessment methods previously described take into account the effects produced by the movement of the yield curve on the cash flows. Thus, if the value of the bond is determined using these methods, the result obtained for the life takes into account both the updating with many interest rates, and the eventually changes of the cash flows. If the life is computed as such, is called *effective duration* or *option-adjusted duration*.

The difference between *modified duration* and *effective duration* for bonds with included options can be, sometimes, very large. So that, for the bonds with included options, one considers that the actual life is the best measure of the sensitivity of price to the parallel movements of the yield curve.

Starting from the theoretical form of the price of a bond:

$$P=\frac{C}{1+y}+\frac{C}{(1+y)^2}+...+\frac{C}{(1+y)^n}+\frac{M}{(1+y)^n}$$

where:
P = bond's price;
C = semi-annual coupon (in units);
y = half of the yield until maturity or the requested yield;
n = number of semesters (number of years until maturity * 2);
M = principal reimbursed at maturity.

One can determine the approximate change of the price in the case of small changes of yield [3], computing the 1st degree derivative of the previous equation in terms of the yield. That allows us to identify the computing formula for *the Macauly duration* indicator:

$$Macaulay\ duration=\frac{\sum_{t=1}^{n}\frac{tC}{(1+y)^t}+\frac{nM}{(1+y)^n}}{P}$$

and the relation between this and the modified duration:

$$Modified\ duration=\frac{Macaulay\ duration}{1+y}$$

There is presented in Table 3 an example for the calculation of the two indicators: Macauly duration and modified duration for a bond with 5 years maturity, coupon 5% and 8% yield until maturity.

Table 3
The calculation of Macauly duration and modified duration for the 5%/5 bond, 8% yield until maturity

Period (t)	CF[1] for 100 units par value	PV[2] for 1 unit	PVCF[3]	PVCF[3] pondered with maturity
a	*b*	*c=1/(1+0.04)^t*	*d=b*c*	*e=a*d*
1	2,5	0,96153846	2,4038462	2,4038462
2	2,5	0,92455621	2,3113905	4,6227811
3	2,5	0,88899636	2,2224909	6,6674727
4	2,5	0,85480419	2,1370105	8,5480419
5	2,5	0,82192711	2,0548178	10,2740888
6	2,5	0,79031453	1,9757863	11,8547179
7	2,5	0,75991781	1,8997945	13,2985617
8	2,5	0,73069021	1,8267255	14,6138041
9	2,5	0,70258674	1,7564668	15,8082016
10	102,5	0,67556417	69,2453273	692,4532730
			87,8336563	**780,5447890**

[1]CF = cash flow; [2]PV = present value; [3]PVCF = present value of cash flow.

Macauly duration (in halves of years) = 780.544789 / 87.8336563 = 8.89
Annualized Macauly duration = 8.89 / 2 = 4.45
Annualized modified duration = 4.45 / 1.04 = 4.27

The properties of these two indicators can be seen computing the indicators in a similar way as in the previous example, for the six considered hypothetical bonds:

Table 4
Macauly life and changed life for the six hypothetical bonds

Bond	Macauly duration	Modified duration
8% / 5	4,22	4,06
8% / 20	10,29	9,90
5% / 5	4,45	4,27
5% / 20	11,37	10,93
0% / 5	5	4,81
0% / 20	20	19,23

One can notice that the values obtained for the two indicators are, for all the six bonds, smaller than the maturity. The only case in which the Macauly duration is equal to maturity is the one of the bond with zero coupon, and that can be seen from the formula. The modified duration is, though, smaller than maturity in the case of zero coupon bond as well.

The properties of the modified duration, noticed in the proposed example, are the following:

- The longer the maturity, other factors being constant, the longer the modified duration;
- Generally, the smaller the coupon, other factors being constant, the longer the modified duration;
- The lower the level of yield to maturity, other factors being constant, the longer the modified duration.

4. Convexity

The 2nd degree derivative of the price function is called *the measure of the absolute convexity* and serves for the correction of the approximation of price-yield relation:

$$\text{the measure of the absolute convexity} = \frac{d^2P}{dy^2}$$

And in the case of convexity we can talk about changed convexity (doesn't take into account the potential changes in the cash flows that can occur when the yield is changed) and actual convexity (admits that in the cash flows changes can occur by changing the yield). For the bonds with included options or for the mortgage ones, significant differences between the two types of convexity can occur.

RESULTS AND DISCUSSIONS

The properties of the duration are similar to those of the bond's price volatility, so that the link between the modified duration and price's volatility is direct: the longer the modified duration, the higher the bond's price volatility, making a good measure of the interest rate risk for small changes of the yield.

The drawback is that, no matter an upward or downward movement of the yield curve, the duration approximates the same percentage change of the price, which is not always true. At small changes of the yield, the percentage change of the price is the same, whether the yield increases or decreases, while at large changes of it, that is not so.

Duration offers, in fact, a first approximation for small parallel movements of the yield curve. The approximation can be improved, continuing the process on the second level. The convexity is the additional measuring tool of the bond's price change that supplements the approximation offered by duration.

Duration and convexity give us the amplitude of the price's volatility at yield's changes. The measurement of the interest rate risk pertaining a bond position must take into consideration the expected volatility of the yield. The larger the yield's expected volatility, the larger the interest rate risk for a position with the given life and current value. The measurement of the yield's volatility is made using statistical methods and indicators. The modern theories of portfolio management, developed by *M. Markovitz and J. Tobin,* introduced the concepts of standard deviation and variance, through which one measure how much can an event deviate from the average, the element's volatility respectively. The variance is one of the possibilities to quantify the risk.

CONCLUSIONS

Once measured the interest rate risk for a bond position or for a bonds portfolio, the next step in risk management is the diminishment of exposure to the risk up to an acceptable level. This phase is called risk control.

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