GENERALIZATION OF THE FUNDAMENTAL EQUATION OF THE RESERVOIR

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Abstract

In paper (Vâscu V., Gavrilaş G, Chiorescu E., 2001) a differential equation is derived and solved analytically, under certain simplifying assumptions, as the authors called the fundamental equation of the pool, which is a zero-order model for the dispersion of soluble pollutants in a water body. In this article, a generalized mathematical model is developed and solved numerically using a computer program, by giving up some of those restrictive conditions and playing through a system of two differential equations.

Keywords: dispersion, soluble pollutants, mathematical model, reservoir.

In this article, a generalized mathematical model is designed and numerically solved for the dispersion of soluble pollutants in a water body, playing through a system of two differential equations suitable for a reservoir.

There has been taken into account the work (Vâscu V., Gavrilaş G, Chiorescu E., 2001) about volume control D, which is considered the material, represented by the reservoir tank containing water volume V. As the hypothesis of neglecting the variation of water volume V ($V \approx V_0$ = const.) and accepting a model of zero order, where the concentration depends only on time t, $C = C(t)$, and not considering the domain section D, was derived "*reservoir equation*" or "*pool equation*" (1), which has the first order, is heterogeneous, with variable coefficients:

$$V_0 \frac{dC}{dt} + Q_e C = Q_{mp} + Q_a C_a \qquad (1)$$

where:

$Q_e = Q_e(t)$ - flow of water discharged from the lake [m^3/s];

$Q_{mp} = Q_{mp}(t)$ - pollutant mass flow [g/s];

$Q_a = Q_a(t)$ - refresh water flow with conventional water clean [m^3/s];

$C_a = C_a(t)$ - pollutant concentration in the refreshed water (below the allowable level) [g/m^3].

The initial conditions associated with equation (Y.1.1) are:

$$t = t_0,\ C(t_0) = C_0. \qquad (2)$$

Mass flow of source remediation Q_{mp}, is assessed by the relationship:

$$Q_{mp} = Q_p C_p, \qquad (3)$$

where:

$Q_p = Q_p(t)$ - concentrated source of pollution flow, [m^3/s];

$C_p = C_p(t)$ - pollutant concentration of the concentrated pollution source (beyond permissible level) [g/m^3];

The condition $V \approx V_0$ = constant is satisfied if:

$$Q_a + Q_p - Q_e \cong 0. \qquad (4)$$

With the following assumptions:

- the flow $Q_e(t)$ and concentration $C_a(t)$ are constant:

$$Q_e(t) \equiv Q_{e0},\ C_a(t) \equiv C_{a0};\ (1) \qquad (5)$$

- the flows $Q_{mp}(t)$ şi $Q_a(t)$ varies linearly,

$$Q_{mp}(t) = Q_{mp0} + q_{mp} t, \quad Q_a(t) = Q_{a0} + q_a t, \qquad (6)$$

In paper (1) was particularly drawn the following analytical solution equation (1) with initial conditions (2):

$$C = C(t) = e^{-a(t-t_0)}\left(C_0 - \alpha - \beta t_0\right) + \alpha + \beta t \qquad (7)$$

In the above expression the following notations were introduced:

$$\begin{cases} \alpha = \dfrac{aA - B}{a^2} = \dfrac{Q_{e0}\left(Q_{mp0} + C_{a0}Q_{a0}\right) - V_0\left(q_{mp} + C_{a0}q_a\right)}{Q_{e0}^2} \\ \beta = \dfrac{B}{a} = \dfrac{q_{mp} + C_{a0}q_a}{Q_{e0}} \end{cases} \qquad (8)$$

and

$$a = \frac{1}{V_0} Q_{e0},\ A = \frac{1}{V_0}\left(Q_{mp0} + C_{a0} Q_{a0}\right),\ B = \frac{1}{V_0}\left(q_{mp} + C_{a0} q_a\right) \quad (9)$$

MATERIAL AND METHOD

Below we present the new mathematical model proposed for dispersion of pollutants into the reservoirs .

When the condition $V \approx V_0$ = constant is cannot be imposed on the accumulation of functional considerations, than V = V (t), the mathematical model expressed by relations (1) and (2) had to be reconsidered, resulting a system of two first order differential equations of the addiction variables C and V, and the initial conditions associated with these variables:

$$\begin{cases} \dfrac{\mathrm{d}C}{\mathrm{d}t} = \dfrac{Q_\mathrm{a} C_\mathrm{a} + Q_\mathrm{p} C_\mathrm{p} - Q_\mathrm{e} C}{V} \\ \dfrac{\mathrm{d}V}{\mathrm{d}t} = Q_\mathrm{a} + Q_\mathrm{p} - Q_\mathrm{e} \end{cases} \quad (10)$$

$t = t_0$, $C(t_0) = C_0$ şi $V(t_0) = V_0$. (11)

Generally, the system (10) of initial conditions (11) can be solved only by numerical methods (eg Ruge-Kutta of 5th order). To this end, a computer program, named **Lac_Acumulare_Zero.m** has been prepared and tested.

The program **Lac_ Acumulare_Zero.m**. was prepared based on the mathematical model presented in § 2, and it can handle three types of problems indicated in the ***Menu*** KodP indicator (Listing 1).

Input and output data are indicated in the listing program found in ***Notations Section.***

Listig 1

- % ***Lac_* Acumulare_Zero*.m;***
- % System of differential equations solving program (10);
- % Solve the system of differential equations (10), with initial conditions (11);
- % with MATLAB SUBPROGRAM MATLAB ***ode45.m;***
- % Subprogram call function. ***Func_Disp_Lac.m***
- % to assess the member functions of two system (10), functions which are addicted by variable time t, and addiction variables C and / or V
- % **MENU**
- % KodP = indicator for the type of solved problem;
- % KodP=1; concentration is calculated at the times required;
- % KodP=2; duration is calculated needed to achieve a concentration required;
- % KodP=3; the refresh/discharge flow is calculated necessary to maintain a minimum concentration required;
- % KodS = operating scenario indicator accumulation
- % KodS=1 ; Q_e flow is required;
- % KodS=2 ; Q_e flow is determined by the level condition / constant volume.

Notations

- % **A. Input data**
- %
- % ***0. General program parameters***
- % N_{ec}= number of addiction variables $N_{ec} \in \{1, 2\}$
- % Δt = time step variation, [hours]: (only for KodP=1);
- % ***1. Geometrical characteristics of accumulation***
- % $H_{\_MN}$= water level in the reservoir,, [mdMN]
- % $V_{\_E}$= volume of water sthoursd properly $H_{\_MN}$, [m.c.]
- %
- % ***2. Hydraulic characteristics of accumulation***
- % $t_{\_Qp}$= flow monitoring moments of pollution source, [hours]
- % Q_{p_E}= flow source of pollution, monitored at the time $t_{\_Qp}$, [m.c/s]
- % $t_{\_Qa}$= flow monitoring moments supply, [hours]
- % Q_{a_E}= flow source of pollution, monitored at the time $t_{\_Qa}$, [m.c/s]
- % $t_{\_Qe}$= flow monitoring moments discharged from dam, [hours]: (for KodS=1)
- % Q_{e_E}= discharged flow in the dam section, at $t_{\_Qe}$,[m.c/s] : (for KodS=1)
- %
- % ***3. Qualitative characteristics of the fluency/ influx water in/ from the accumulation***
- % $t_{\_Cp}$= monitoring moments of the source of pollution concentration, [hours]
- % C_{p_E}= source of pollution concentration, monitored at the time $t_{\_C}$, [g/m.c]
- % $t_{\_Ca}$= monitoring moments of the source of supply, [hours]

% C_{aE}= concentration of power supply, monitored at $t_{\_Ca}$, [g/m.c] %

- %
- % ***4. Primary and/or final terms***
- % t_0 = initial time, [hours];
- % C_0 = initial concentration of the lake, [g/m.c];
- % V_0 = the volume of the lake at the moment t_0, [m.c];

- % C_f = final concentration value (for KodP=2) or the concentration value that has to be maintained (for KodP=3);
- % t_{Max}= final moment calculation [hours]: (only for KodP=1);
- %
- % **B. Output Data**
  %
- % tC = calculation time [hours]: (only for KodP=1);
- % Cc = concentration value at the moment tC [g/m.c]: (only for KodP=1);
- % Vc= volume of water in the lake at the moment tC, [m.c]:(for KodP=1 and KodS=1);
- % Hc= level of water in the lake at the moment tC, [m.c]:(for KodP=1 and KodS=1);
- % tMax = when de concentration has value concentration required [hours]: (for KodP=2)%
- % Q0min = minimum value of the refresh / discharge flow [m.c/s]: (for KodP=3)
- % t_Qe= flow monitoring moments discharged from dam, [hours]: (for KodS=2)
- % Qe_E= flow discharged from dam, at t_Qe , [m.c/s]: (for KodS=2)

Lac_ Acumulare_Zero.m. Testing program

Lac_Acumulare_Zero.m program was tested using the analytical solution (7) for this purpose a test example was developed, with maximum generality of the analytical solution (7), in (5) and (6) relations, as being imposed the conditions below:

$Q_{e\,0} \neq 0$, $C_{a\,0} \neq 0$, $Q_{mp\,0} \neq 0$, $q_{mp} \neq 0$,
$Q_{a\,0} \neq 0$ şi $q_a \neq 0$. (12)

Numerical values introduced in relations (7)÷(9) are as follow:

t_0 =0, C_0 =0.26, V_0=3710000
Q_{mp0}= 59.999999999999901, q_{mp} =17.5,
Q_{p0}= 0.50 (13)
Q_{a0} = 50, q_a =-0.1, Q_{e0} = 50.5000, C_{a0} =0.1

In evaluating expressions (9) and, respectively (8), has been resulted gradually:

a = 1.3612e-5, A = 1.7520e-5, B = 1.3095e-9 and

α = -5.7806, β = 9.6205e-5.

The above numerical values of the analytical solution (7) have the following particular expression:

$$C = \mathrm{C}(t) = 6.0406 \cdot e^{5.7806 \cdot t} - 5.7806 + 9.6205e^{-5} \cdot t \quad (14)$$

Requiring calculation step Δt =1 şi t_{Max}=45, for the analytical solution (14) has been resulted the following table of values for computing times t_{c_a} and the concentration of C_{c_a}, from this moments:

t_{c_a} = [0 1 2 .... 44 45]
C_{c_a} =[0.2600 0.3175 0.3887 0.4732 0.5701 0.6790 0.7993
0.9303 1.0717 1.2228 1.3833 1.5527 1.7305 1.9164
2.1100 2.3108 2.5187 2.7331 2.9539 3.1806 3.4131
3.6511 3.8942 4.1422 4.3950 4.6522 4.9137 5.1792
5.4486 5.7217 5.9983 6.2782 6.5613 6.8474 7.1364
7.4282 7.7225 8.0194 8.3186 8.6201 8.9237 9.2293
9.5369 9.8464 10.1576 10.4705]

According to the input data (13), in the program **Lac_Acumulare_Zero.m** has been introduced the following numerical data:

Menu indicators :KodP = 1, KodS = 2;

0. The general Parameters of the program
N_{ec} = 1;
1. Geometrical characteristics of accumulation:
$H_{\_MN}$ = 210.50, $V_{\_E}$ =3710000;
2. Hydraulic characteristics of accumulation:
$t_{\_Qp}$ = $t_{\_Qa}$ = [0 5 10 15 20 25 30 35 40 45];
Q_{p_E} =[0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00 4.50 5.00];
Q_{a_E} = [50.00 49.50 49.00 48.50 48.00 47.50 47.00 46.50 46.00 45.50];
3. Qualitative characteristics of the fluency/ influx water in/ from the accumulation: $t_{\_Cp}$ = $t_{\_Ca}$ = [0 5 10 15 20 25 30 35 40 45];
C_{p_E} =[150.0 152.5 155.0 157.5 160.0 162.5 165.0 167.5 170.0 172.5];
C_{a_E} = [0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10];
4. Primary and/or final terms:
t_0 = 0, C_0 = 0.2600, V_0 = 3710000, t_{Max} = 45.

RESULTS AND DISCUSSIONS

Out of running the program **Lac_Govora.m** has been resulted:

t_C =[0 0.7742 1.5485 2.3227 3.0969 4.221 5.346 6.4719 7.5969 8.7219 9.8469 10.9719 12.0969 13.2219 14.3469 15.4719 16.5969 17.7219 18.8469 19.9719 21.0969
22.2219 23.3469 24.4719 25.5969 26.7219 27.8469 28.9719 30.0969 31.2219 32.3469 33.4719 34.5969 35.7219 36.8469 37.9719 39.0969 40.2219 41.3469 42.4719 43.5969 43.9477 44.2985 44.6492 45.0000]

C_c=[0.2600 0.3137 0.3741 0.4411 0.5144 0.6318 0.7616 0.9033 1.0564 1.2204 1.3949 1.5794 1.7735 1.9769 2.1891 2.4099 2.6389 2.8758 3.1203 3.3722 3.6312 3.8970 4.1694 4.4482 4.7332 5.0242 5.3209 5.6233 5.9312 6.2443 6.5626 6.8859 7.2140 7.5469 7.8844

8.2265 8.5729 8.9236 9.2786 9.6376 10.0007 10.1147 10.2291 10.3439 10.4590]
$t_{\_Qe}$ = [0 5 10 15 20 25 30 35 40 45];
Q_{e_E}=[50.50 50.50 50.50 50.50 50.50 50.50 50.50 50.50 50.50 50.50];
Analytical solution, reproduced by the set of points:

$$\left(t_{C\_a}(i), C_{C\_a}(i)\right), i = 1, 2, \ldots, 46, \qquad (15)$$

is represented in *fig.1*., by continuous line.

Numerical solution, reproduced by the set of points

$$\left(t_C(i), C_C(i)\right), i = 1, 2, \ldots, 46, \qquad (16)$$

is represented in *fig.1*., by marker ‘ * ’.

It can be find a good match between numerical and analytical solutions - which shows that the **Lac_Acumulare_Zero.m** development programwas sufficiently reliable

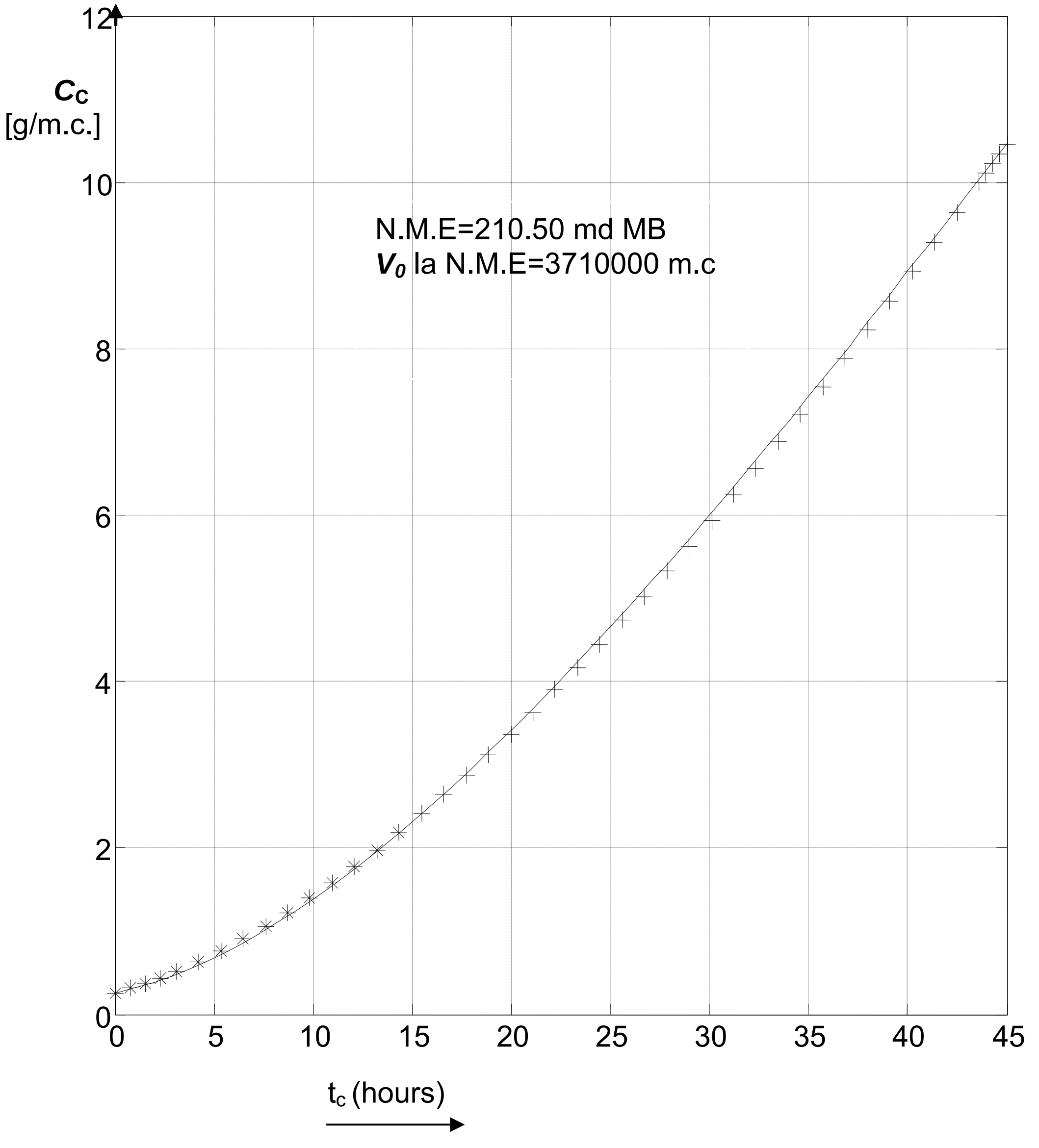

Figure 1 **Variation of concentration in the lake Govora scenario at constant level**
Comparison between analytical solutions (continuous line) and numerical (marker „* ")

CONCLUSIONS

The conception and numerical solution of a generalized mathematical model for the dispersion of soluble pollutants in a body of water has led to satisfactory results with sufficient precision to any lake met in hydrotechnic practice.

This mathematical model is used to predict pollutant concentration variation over time, being able to take such measures necessary to protect or minimize the negative effects.

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