THE STUDY OF ICE FORMATION IN FREEZING PROCESSES OCCURING IN PLANE GEOMETRIES

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Abstract

Food freezing processes are largely studied because of their importance in achieving food safety. In plate freezers, the cold surfaces in contact with the product are stainless steel or aluminum hollow plates through which circulates the refrigerant. The rate of the freezing process is critical to the product's quality and to the productivity of the process and therefore the freezing dynamics is of extreme importance. Another application may be the production of ice blocks required for products refrigeration in places where no refrigeration facilities are available. The aim of the paper is to study theoretically the process of ice buildup in the space between the parallel plates of a plate freezer. This study is performed using implicit finite difference schemes. The method involves the use of variable step networks attached to the liquid and solid domains. The evolutions of the freezing rate and of the solid layer thickness are determined for different geometry ratios and cooling temperatures.

Key words: freezing, freezing rate, finite differences, implicit scheme, variable step network.

Mathematical modeling of the freezing process is necessary to make accurate predictions regarding the process dynamics, i.e. its time-dependent evolution. Knowing the dynamics of freezing will allow the correct calculation of the freezing equipment in terms of performance parameters that it must ensure, so as to be able to conduct freezing under optimal conditions.

Freezing is used in food industry to preserve raw or processed food products. Among many methods, on site freezing is used in plate freezers in the case of freshly harvested fruits, vegetables, offal, fish or seafood (Ansari, F.A., 1984). Plane geometry also intervenes when making ice blocks.

The main parameters that control the dynamics of the process are (Ansari, F.A., 1984), (Ansari, F.A., 1999):

- the initial temperature of the product;
- the product's geometry (size, surface and shape);
- the product's structure (homogeneous or heterogeneous);
- the product's thermal properties;
- the freezing temperature.

Heat transfer processes occurring during freezing are of high complexity, because on one hand the product has complicated shape and structure, and on the other hand, the thermal regime is by definition a non-stationary one and it implies the phase change of water (freezing). The mathematical model of these processes refers to the moving boundary problem, i.e. the unsteady conduction heat transfer in which the solid-liquid phase change occurs and the solid-liquid interface moves through the phase change material.

The moving boundary problem in the case of phase change was subjected to many approaches (Baird C.D., Gaffney J.J., 1976), (Crank J., Gupta R.S., 1980), (Hale N.W., Viskanta R., 1980) (Ozisik M.N., 1994). The one-dimensional case allows us to use the finite difference method and thus obtain the numerical solution of the moving boundary problem by transposing the heat transfer partial differential equation in the finite difference form and to solve it via a computer code.

MATERIAL AND METHOD

Let us consider a rectangular metallic container filled with water (phase change material – PCM). In order to assume the heat transfer as one-dimensional, let us suppose that the depth of the container is much smaller with respect to its height and width. At the beginning, the entire system (metallic wall + water) has the same temperature T_0, above the fusion temperature T_F of the PCM. The container is cooled on its both sides and consequently one can consider only half of its depth. At time $\tau = 0$, the wall is cooled (at temperature $T_C < T_F$) from the outside. Consequently, the perturbation propagates through the wall until it reaches its inner surface (phase I). Phase II is represented by the cooling of the liquid until the temperature of the inner surface of the wall equals T_F. At this moment, phase III (solidification) starts and consequently the first ice layer grows. Its thickness increases as time passes, whereas the interface moves across the PCM region.

The physical properties of the wall and of the PCM are supposed constant.

By attaching a constant step network to the metallic wall (N_W – number of nodes) and a variable step one to the two PCM domains (liquid – water: N_L nodes, and solid – ice: N_S nodes), one can write the finite difference equations that describe the heat transfer phenomena that occur. The system is schematized in *Figure 1.*

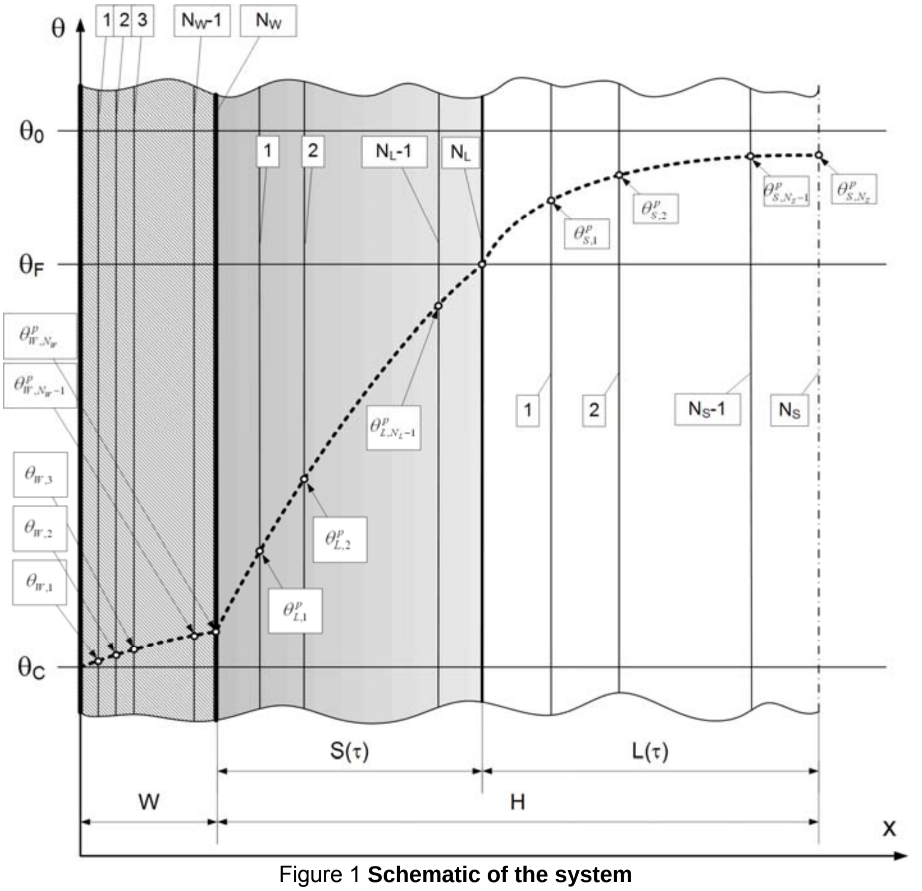

Figure 1 **Schematic of the system**

Using as variable the dimensionless temperature defined as $\theta = (T - T_C) / (T_0 - T_C)$, the finite difference equations describing the unsteady one-dimensional heat transfer with phase change are (Horbaniuc, B., 1996; Incropera, F.P., De Witt, D.P., 2002, Ozisik, M.N., 1994):

The heat transfer equation for a constant depth domain (such as the metallic wall, or the liquid domain before freezing starts) is:

$$-\theta_{m-1}^{p} + \left(2 + \frac{1}{\alpha}\right)\theta_{m}^{p} - \theta_{m+1}^{p} = \frac{1}{\alpha}\theta_{m}^{p-1} \qquad (1)$$

where m is the current node, α is a parameter that dictates the stability and convergence of the finite difference scheme, and p is the number of the time step. This equation corresponds to the so-called implicit scheme that links the actual temperatures in three consecutive nodes to the temperature in the current node at the preceding time step *p – 1*. This type of scheme is unconditionally stable and convergent regardless of the magnitude of the time step $\Delta\tau$, but it requires the simultaneous resolution of all of the equations for all of the nodes. We have surmounted this difficulty by applying the Gauss elimination technique.

During freezing, the depth of a PCM domain is variable (the interface is moving) and therefore the space step is variable too. For this reason, the heat transfer equation in the finite difference form is:

$$-\theta_{m-1}^{p} + \left(2 + \frac{1}{\alpha}\right)\cdot\theta_{m}^{p} - \theta_{m+1}^{p} = \frac{1}{\alpha}\cdot\tilde{\theta}_{m}^{p-1} \qquad (2)$$

where $\tilde{\theta}_{m}^{p-1}$ is the temperature in node *m* at the preceding time step p – 1, but in the position of the node corresponding to the new network (during the current time step *p*, the node has migrated from its previous position to the new one). This situation is illustrated by *Figure 2*.

In order to make the equation match the actual evolution of the phenomenon, one needs to find the value of the nodal temperature $\tilde{\theta}_{m}^{p-1}$. Heitz and Westwater have solved the problem by linearly extrapolating the values of the nodal temperatures at time (p – 1) $\Delta\tau$ (Heitz, W.L., Westwater, J.W., 1970). We have adopted a more accurate solution by using Lagrange polynomials (Horbaniuc, B., 1996).

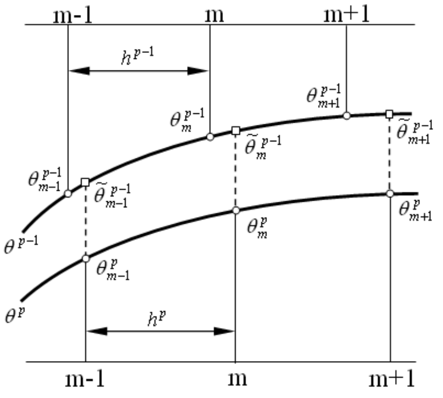

Figure 2 **Migration of the nodes during a time step**

The equation of the interface rate is:

$$S^p = \frac{T_0 - T_C}{2 \cdot \rho \cdot l_f} \cdot \left[\frac{\lambda_S \cdot N_S}{S^p} \cdot \left(\theta^p_{S,N_W+N_S-2} - 4 \cdot \theta^p_{S,N_W+N_S-1} + 3 \cdot \theta_F \right) + \frac{\lambda_L \cdot N_L}{H - S^p} \cdot \left(3 \cdot \theta_F - 4 \cdot \theta^p_{L,N_W+N_S+1} + \theta^p_{L,N_W+N_S+2} \right) \right] \tag{3}$$

where □ is the density of ice, l_f is the latent heat of fusion of water, *S* is the depth of the solid domain, and □ is the thermal conductivity. Subscripts account as follows: *W* for the metallic wall, *S* for the solid phase (ice) and *L* for the liquid phase (water).

By applying the finite difference equations one obtains two sets of linear algebraic equations (written in the matrix form), one for the wall – ice domains:

$$C_1 \cdot \Theta_1 = B_1 \tag{4}$$

and one for the liquid domain:

$$C_2 \cdot \Theta_2 = B_2 \tag{5}$$

where C_1 and C_2 are the diagonal coefficient matrices, □$_1$ and □$_2$ the unknown (temperature) column vectors and B_1 and B_2 are column vectors:

$$C_1 = \begin{pmatrix}
\mu_1 & -\mu_2 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
-1 & \sigma_W & -1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & -1 & \sigma_W & -1 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & \ldots & -1 & \sigma_W & -1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & -(3-y) & 3\sigma_W - 4y & -4z & z & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & y & -4y & 3\sigma_j - 4z & -(3-z) & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & -1 & \sigma_j & -1 & \ldots & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & -1 & \sigma_j & -1 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & -1 & \sigma_j
\end{pmatrix} \tag{6}$$

$$\Theta_1 = \begin{pmatrix} \theta^p_{W,1} \\ \theta^p_{W,2} \\ \theta^p_{W,3} \\ \ldots \\ \theta^p_{W,N_W-2} \\ \theta^p_{W,N_W-1} \\ \theta^p_{j,N_W+1} \\ \theta^p_{j,N_W+2} \\ \ldots \\ \theta^p_{j,N_W+N_S-2} \\ \theta^p_{j,N_W+N_S-1} \end{pmatrix} \tag{7}$$

$$B_1 = \begin{pmatrix} v \\ b_{W,2} \\ b_{W,3} \\ \ldots \\ b_{W,N_W-2} \\ 3b_{W,N_W-1} \\ 3b_{j,N_W+1} \\ b_{j,N_W+2} \\ \ldots \\ b_{j,N_W+N_S-2} \\ b_{j,N_W+N_S-1} + \theta_T \end{pmatrix} \tag{8}$$

$$C_2 = \begin{pmatrix} \sigma_L & -1 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ -1 & \sigma_L & -1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & -1 & \sigma_L & -1 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & \ldots & -1 & \sigma_L & -1 & \ldots & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & -1 & \sigma_L & -1 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & -1 & \frac{3\sigma_L - 4}{2} \end{pmatrix} \quad (9)$$

$$\Theta_2 = \begin{pmatrix} \theta^p_{L,1} \\ \theta^p_{L,2} \\ \theta^p_{L,3} \\ \ldots \\ \theta^p_{L,m} \\ \ldots \\ \theta^p_{L,N_L-2} \\ \theta^p_{L,N_L-1} \end{pmatrix} \quad (10) \qquad B_2 = \begin{pmatrix} b_{L,1} + \theta_T \\ b_{L,2} \\ b_{L,3} \\ \ldots \\ b_{L,m} \\ \ldots \\ b_{L,N_L-2} \\ 3b_{L,N_L-1}/2 \end{pmatrix} \quad (11)$$

The two sets are coupled via the interface rate equation (3).

Both sets have been solved by means of the Gauss elimination technique.

RESULTS AND DISCUSSIONS

We have considered the case of water initially at T_0 = 15°C (15 degrees of superheating). The metallic wall was supposed to consist of a 1 mm thickness steel sheet. The depth of the PCM domain has been considered in three cases: 15 mm, 30 mm and 50 mm, corresponding to a wall to PCM ratio W/H of 0.066, 0.033, and 0.020 respectively. The outer surface of the wall is cooled at different temperatures (–20, –25, and –30°C) which implicitly means that the boundary condition is of the first type. The algorithm modeled the three phases of the process: propagation across the wall, propagation across the superheated liquid, and solidification. We have studied the evolution of the interface rate and of the solid fraction (the percentage of the solid phase depth with respect to the total depth of the PCM domain) versus the dimensionless time $\overline{\tau}$ defined as:

$$\overline{\tau} = \frac{a_S \tau}{H^2} \quad (12)$$

where a_S is the thermal diffusivity of the solid phase, τ is time and *H* is the total depth of the PCM. The results are shown in *Figures 3* through *6*.

Figure 3 plots the interface rate $\dot{S}$ versus $\overline{\tau}$ for a value of the cooling temperature (T_C = –25°C) in the case of the three values of the W/H ratio. One notices that the higher values of the interface rate characterize the thin domain (W/H = 0.066), whereas the slower evolution can be observed when the domain is thicker (W/H = 0.020). At lower W/H ratios, the plot exhibits an inflection point which fades out as the ratio increases.

Figure 4 comprises the interface rate plots for all of the W/H ratios and cooling temperatures. The trends that have been noticed for a single cooling temperature (see *Figure 3*) can also be seen at different W/H ratios. The highest interface rates are found when W/H is less and T_C is lower. The higher the W/H ratio and the higher the cooling temperature T_C, the slower the process is (low interface rate).

The ice fraction evolution is plotted in *Figure 5* for W/H = 0.020 and for different cooling temperatures. As expected, the slowest evolution corresponds to the highest cooling temperature and the fastest one, to the lowest temperature.

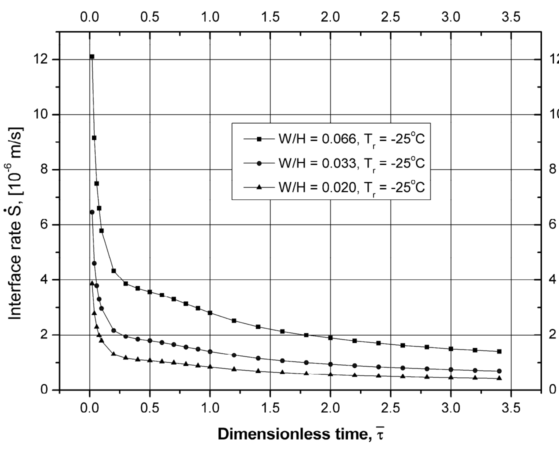

Figure 3 **The interface rate versus the dimensionless time at T_C = – 25°C for different W/H ratios**

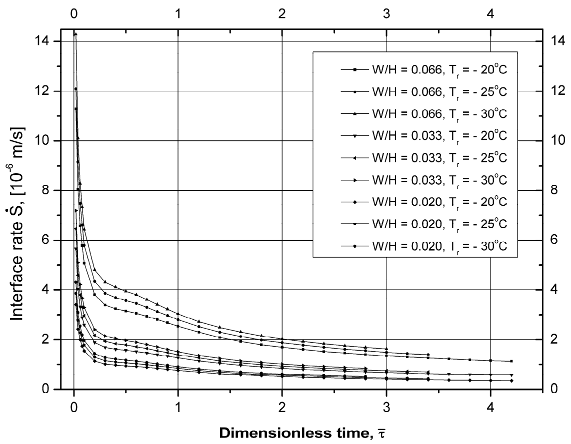

Figure 4 **The interface rate versus the dimensionless time for all of the considered W/H ratios and cooling temperatures**

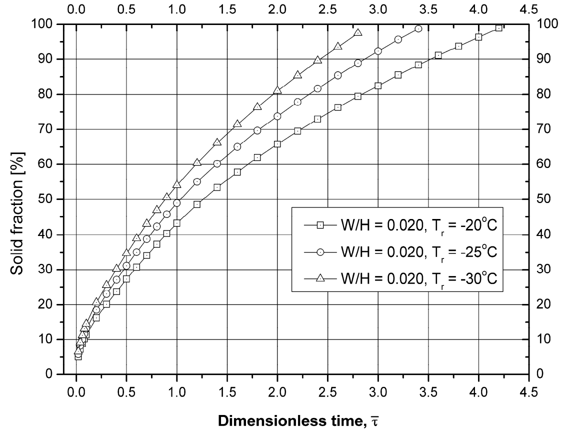

Figure 5 **Solid fraction versus the dimensionless time at W/H = 0.020 for different cooling temperatures**

The evolution of the solid fraction for different W/H ratios is plotted for the case when the cooling temperature is – 25°C (*figure 6*).

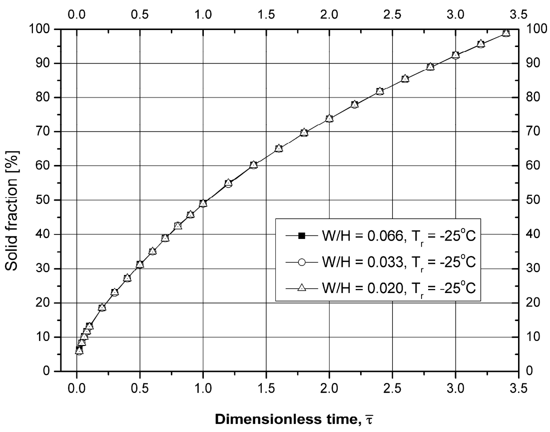

Figure 6 **Solid fraction versus the dimensionless time at T_0 = –25°C for different values of the W/H ratio**

Unlike the interface rate plots, the curves in this case overlap perfectly. This means that at a certain degree of cooling of the metallic wall the depth of the PCM domain practically has no influence on the growth of ice layer, the only parameter that dictates the evolution of the solid fraction being the cooling temperature.

CONCLUSIONS

Knowledge on the dynamics of the freezing process is useful when designing food freezing equipment or ice blocks making equipment.

The freezing process in one-dimension heat transfer in plane geometries has been studied by means of the implicit finite difference scheme using a variable step network attached to the PCM domain (water in this case). In order to surmount the difficulty caused by the node migration in terms of the nodal temperature at the current time step with respect to the previous position of the respective node, we have used the Lagrange polynomials interpolation technique.

The numerical results obtained on an example that considered a steel wall cooled at different temperatures and a PCM domain of different thicknesses lead to the following conclusions with respect to the dynamics of the freezing process:

– the thicker the PCM domain, the lower is the interface rate for a given cooling temperature;
– the lower the cooling temperature, the higher is the interface rate for a given thickness of the PCM domain;
– the ice fraction grows more rapidly at lower cooling temperatures at the same W/H ratio;
– the W/H ratio has no influence on the solid fraction at the same cooling temperature.

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