VALIDATION OF THE MODEL OF A MILKING MACHINE VACUUM SYSTEM BY THE MEANS OF TEATCUP FALL-OFF TESTS

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Abstract

A previous paper presented some preliminary researches regarding the use of a variable frequency driver (VFD) in order to maintain a constant vacuum in a milking system, by adjusting the speed of the vacuum pump; the PID controller used by the vacuum regulating system was tuned by the method of trial and error, rather than as a result of a theoretical study. In order to proceed to a more systematic approach to this solution for vacuum regulation a mathematical model of the vacuum system should be developed. In the present paper a mathematical model is presented; the model is used to study the dynamic behavior of the milking system and it assumes that the system consists of a single air tank, provided with a vacuum pump port and an air-using port. In order to validate the model and study the system's response to vacuum variation due to a pulse air leak the detachment (fall-off) of one teatcup was simulated; the teatcup was detached for 10, 20 and 30 seconds respectively. During the fall-off tests the rate of air flow into the system was measured by the means of a rotameter and the vacuum level was recorded. The experimental results were compared with the ones predicted by the model and it was concluded that the model accurately describes the response of the system.

Key words: variable frequency drive, vacuum system model, teatcup fall-off

INTRODUCTION

The vacuum level and stability have a major effect over the mechanical milking process. According to Bade, Reinemann, Zucali, Ruegg and Thompson [1], the vacuum level and the compressive load applied to the teat when the liner collapses are the factors affecting the peak milk flow rate. Cows have a biological limit for a positive reaction to vacuum and exceeding these limits may lead to damage of the teat tissue or slipping of milking clusters off the teat, resulting in an extended milking time and in improper milking [3]. Vacuum fluctuations generated within the milking cluster may lead to direct bacterial penetration, thus causing mastitis [3].

In a previous paper [4] a variable frequency driver (VFD) was used in order to control the vacuum pump of the milking system, aiming to make the vacuum pump draw only the amount of air needed to compensate the air entering the system through pulsators, claws, leaks. Figure 1 presents the basic layout of a milking system with a VFD controlled vacuum pump.

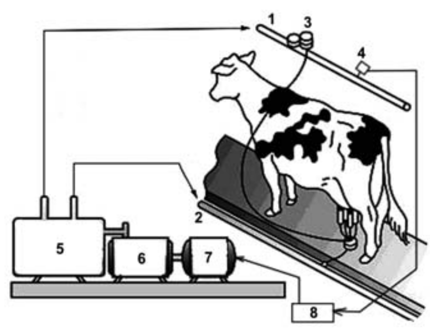

Fig. 1 Basic layout of a milking system with a VFD controlled vacuum pump [4]
1-permanent vacuum pipeline; 2-milk pipeline; 3-pulsator; 4-pressure sensor; 5-receiver; 6-vacuum pump; 7-electric motor; 8-VFD.

The experimental tests confirmed the validity of the idea and showed that vacuum regulation by the means of the VFD

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The manuscript was received: 26.06.2014
Accepted for publication: 17.07.2014

controlled vacuum pump did not adversely affect the working parameters of the system, while achieving better results regarding the stability of the permanent vacuum.

In order to proceed to a more systematic study of this solution for vacuum regulation a mathematical model of the vacuum system should be developed. In the present paper a mathematical model is presented; the model is used to study the dynamic behavior of the milking system and it assumes that the system consists of a single air tank, provided with a vacuum pump port and an air-using port.

MODELING THE MILKING MACHINE VACUUM SYSTEM AS A FIRST ORDER SYSTEM

A first order system, as shown in fig. 2, has an input variable x (X) and an output variable y (Y)[2]; s is the Laplace independent variable and t is the time variable [2].

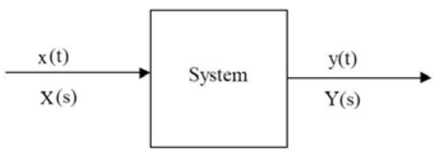

Fig. 2 Schematics of a first order system

Laplace transforms allow solving of linear differential equations describing a dynamic process; the Laplace transform converts the differential equations in time *t* into algebraic equations in complex variable *s*. The Laplace transform of a general function of time f(t) is defined as:

$$£[f(t)] = F(s) = \int_0^\infty f(t) \cdot e^{-s \cdot t} dt \cdot \qquad (1)$$

The transfer function G(s) describes the effect of system input over the system output in the Laplace domain:

$$G(s) = \frac{Y(s)}{X(s)} \cdot \qquad (2)$$

The standard form of a first-order transfer function is:

$$G(s) = \frac{K}{1 + \tau \cdot s}, \qquad (3)$$

where K is the steady-state gain of the system (the ratio of the output to the input at $t = \infty$) and τ is the time constant of the system (the time required for the output to reach 63.2% of the steady-state value – fig. 3).

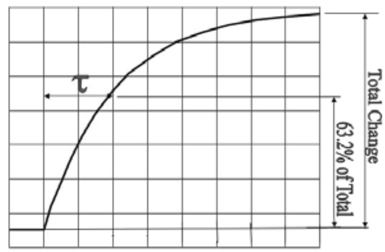

Fig. 3 Time constant of a first order system

For the case of the mechanical milking system, the vacuum system is considered to be composed of a single air tank, provided with a vacuum pump port and an air-using port [5, 6], as shown in fig. 4; $\dot{m}_1$ represents the mass airflow rate of the vacuum pump and $\dot{m}_2$ is the mass airflow rate into the system

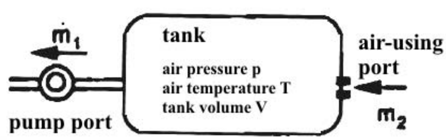

Fig. 4 Schematics of the milking system [5]
$\dot{m}_1$, $\dot{m}_2$ -mass air flow rate

The following equations may be written [6]:

$$\frac{dM}{dt} = \dot{m}_2 - \dot{m}_1 = \dot{m}_2 - q \cdot \frac{M}{V}, \qquad (4)$$

$$p = R \cdot T \cdot \frac{M}{V}, \qquad (5)$$

where M is the mass of air in the air tank, V is the tank volume, q is the volumetric flow rate of the vacuum pump [m^3/s], R is the gas

[2] y(t)=f{x(t0}; Y(s)=f'{X(s)}.

constant for air (R=287 J/kg·K) and T is the air temperature [K].

Using equations (1) and (2) the system transfer function becomes [Tan et al]:

$$G(s) = \frac{p(s)}{\dot{m}_2(s)} = \frac{R \cdot T / q}{1 + s \cdot V / q}. \quad (6)$$

Fig. 5 presents the system response when the air flow rate increases due to the detachment of one teatcup: when the mass flow rate $\dot{m}$ increases by $\dot{m}_p$ the absolute system pressure p_n increases by p_p.

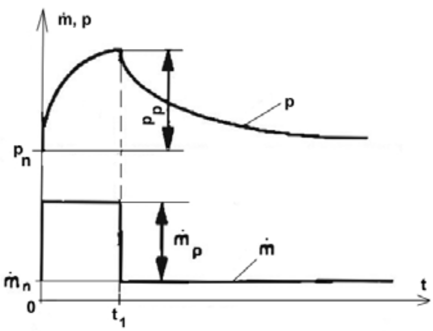

Fig. 5 Model response to mass airflow rate variation [6]
p-absolute pressure; t_1-detachment duration.

The mass flow rate resulting from the pulse air leak $\dot{m}_p$ is [6]:

$$\dot{m}(s) = \frac{\dot{m}_p}{s} - \frac{\dot{m}_p}{s} \cdot e^{-t_1 \cdot s}. \quad (7)$$

Introducing equation (7) into equation (6) and applying the inverse Fourier transform finally leads to:

$$p(t) = \frac{R \cdot T}{q} \cdot \dot{m}_p \cdot \begin{bmatrix} \Phi(t) - \Phi(t - t_1) - e^{-q \cdot t / V} + \\ + \Phi(t - t_1) \cdot e^{-q \cdot (t - t_1)/V} \end{bmatrix},$$

where Φ(t) is the step function, defined as follows:

$$\Phi(t) = \begin{cases} 0, & \text{if} \quad t < 0 \\ 1, & \text{if} \quad t \geq 0 \end{cases}.$$

MATERIAL AND METHOD

A bucket milking machine was used in this study; fig. 6 presents the schematics of the milking machine. The machine was equipped with a BRK pneumatic type pulsator and four Boumatic R-1CX type teatcups. During the test the milking system was operated with a vacuum level of 40.5 kPa, regulated by the means of the mechanical vacuum regulator (3).

The characteristics of the vacuum system were:

- vacuum pump volumetric flow rate $q=4.69 \cdot 10^{-3}$ [m^3/s];
- tank volume $V=3.5 \cdot 10^{-2}$ [m^3].

The air temperature was T=293 [K].

In order to validate the model and study the system’s response to vacuum variation due to a pulse air leak the detachment (fall-off) of one teatcup was simulated; the teatcup was detached for 10, 20 and 30 seconds respectively. During the fall-off tests the rate of air flow into the system was measured by the means of a rotameter (fig. 7) and the evolution of the vacuum level was recorded using the pressure sensor (4, fig. 6).

The air flow rate into the system during the fall-off test was $\dot{m}_p = 7.6 \cdot 10^{-5}$[kg/s] (average value).

The steady state gain K of the model and the time constant τ were calculated with the relations [6]:

$$K = \frac{R \cdot T}{q}, \quad \tau = \frac{V}{q}.$$

For the milking system taken into account the following values were obtained:

- $K=1.79 \cdot 10^5$ [kPa·s/kg];
- τ=7.47 [s].

Using the experimental data the system steady state gain K_s and the time constant τ_s were evaluated. The system steady-state gain was calculated with the formula:

$$K_s = \frac{\Delta p}{\dot{m}_p},$$

where Δp is the vacuum drop when the teatcup is detached.

The time constant τ_s was considered to be the time required for the output vacuum to reach 63.2% of the final value when the teatcup was detached.

RESULTS AND DISCUSSION

Fig. 8, 9 and 10 present the experimental results of the fall-off tests; the model data ("model") and data from three experimental replicates ("experiment 1", "experiment 2", "experiment 3") are shown on each chart.

The tests clearly show that there are only small differences between model and experimental data and that the curves corresponding to the experimental data follow closely the theoretical curves predicted by the model.

In order to evaluate the differences between the experimental data and model data fig. 11, 12 and 13 present the ±2.5% y errors bars superposed over the model curve.

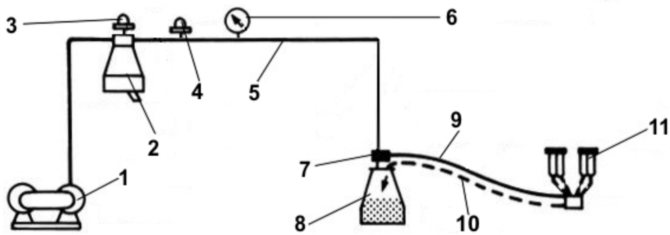

Fig. 6 Schematics of the milking machine [4]
1-vacuum pump; 2-interceptor; 3-vacuum regulator; 4-pressure sensor; 5-vacuum line; 6-vacuum gauge; 7-pulsator; 8-bucket; 9-long vacuum line; 10-long milk line; 11-milking unit

Fig. 7 Rotameter and teatcups

The charts show that the majority of the experimental data were within the ±2.5% variation domain.

Table 1 presents the results concerning the steady-state gain and time constant obtained from the experimental results.

The experimental steady-state gain is with 9% lower than the value given by the model; in the meantime the time constant of the system is with 20% lower than the value predicted by the model.

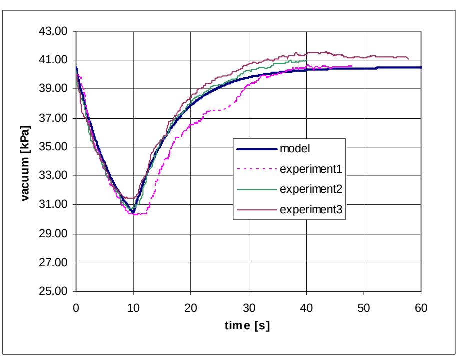

Fig. 8 Experimental and model data for the 10s detachment

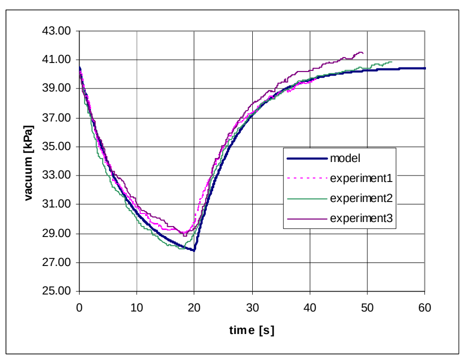

Fig. 9 Experimental and model data for the 20s detachment

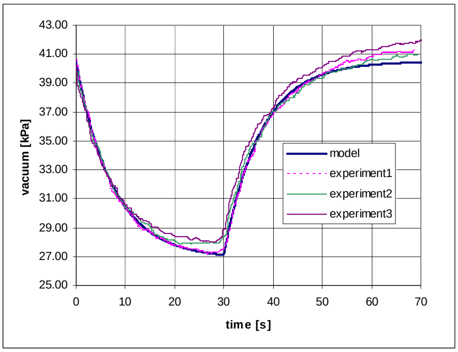

Fig. 10 Experimental and model data for the 30s detachment

Table 1 Experimental results for the time constant and steady-state gain

Teatcup detachment time [s]	Item	
	$K_s \cdot 10^{-5}$ [kPa·s/kg]	τ_s [s]
10	1.34	4.72
20	1.72	5.7
30	1.82	7.6
Average	1.63±0.146	6.00±0.845

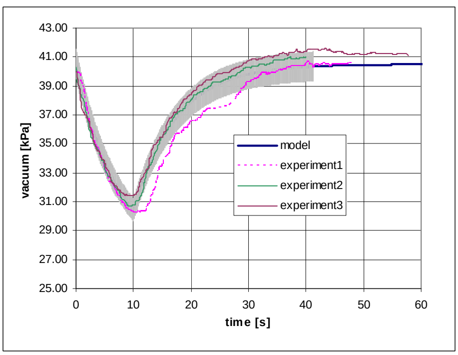

Fig. 11 Model data with ±2.5% error bars, for 10s teatcup detachment

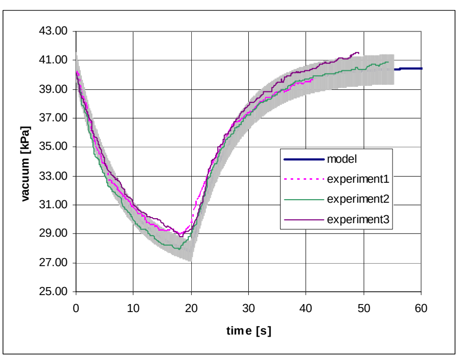

Fig. 12 Model data with ±2.5% error bars, for the 20s teatcup detachment

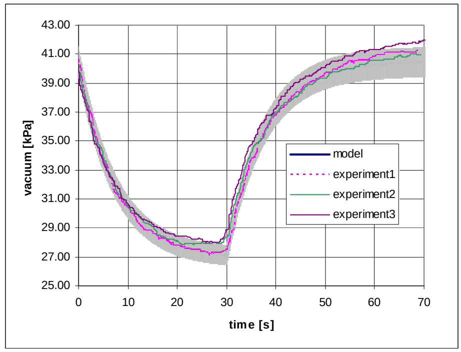

Fig. 13 Model data with ±2.5% error bars, for the 30s teatcup detachment

The inaccuracy of the predicted time constant may be due to the assumptions that air is a perfect gas and that the system is isothermal, with only small variations of the air temperature [5, 6].

If the process is considered adiabatic [6, 7], the time constant is calculated with the relationship:

$$\tau = \frac{V}{\gamma \cdot q},$$

where γ=1.4 is the heat capacity ratio of air.

As a result the time constant of the model becomes τ=5.33 s, a value which is much closer to the average value of 6 s given by the experiments (12,5% lower).

CONCLUSIONS

1. A mathematical model of the mechanical milking system was developed, assuming it is a first order dynamic system consisting of a single air tank, provided with a vacuum pump port and an air-using port.

2. In order to validate the model and study the system's response to vacuum variation due to a pulse air leak the detachment (fall-off) of one teatcup was simulated; the teatcup was detached for 10, 20 and 30 seconds respectively. During the fall-off tests the rate of air flow into the system was measured by the means of a rotameter and the vacuum level was recorded.

3. As a result of the tests it was concluded that the developed model is accurate, the majority of the experimental

values being comprised within the ±2.5% range of the model.

4. However, the assumption that the process is isothermal led to a relatively high difference between the predicted value of the time constant and the value obtained during the experiments. This difference diminished if the adiabatic hypothesis was considered.

5. Developing a more complex model of the milking system is taken into account for a future work, aiming to obtain more accurate predictions.

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