# MATHEMATICAL METHOD FOR FORMULATING ANIMAL FEED RATIONS

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**Abstract**

*The aim of the paper is to present a newly developed model for the optimization of feed rations for active/trained sport horses. The model is structured on two different sub-models, the first of which is based on the linear programming (LP) technique and the second on weighted-goal programming (WGP). The model was tested under two different WGP scenarios to formulate two different feed rations. The results show that feed rations calculated with the modified WGP technique are more balanced (to satisfy all animal nutritional requirements), and that the costs of feed rations are lower compared with LP. The ration was about 10% cheaper and there were no surpluses of any nutrient.*

**Key words**: Feed rations, optimization, linear programming, weighted goal programming

## INTRODUCTION

The two most commonly used mathematical techniques for optimization of feed rations are traditional linear programming (LP) and weighted goal programming (WGP) with penalty functions. LP was established first, in 1951, for solving agricultural problems and calculating feed rations [11]. Optimizing feed rations is important from the economic aspect and health status of animals (animal welfare). Animals used for intensive milk/meat production or intensive sport activity (sport horses) are exposed to stressful situations so often. It is therefore important to calculate the cheapest feed rations which satisfy all nutrition requirements. The LP and WGP methods are highly useful for solving nutritional and other agricultural problems ([1], [2], [4], [6], [7], [8], [10], [12], [13], [14]), but they are more commonly used for animals reared for intensive production.

The main difference between these two mathematical approaches is in the number of goal optimizations. With LP, the user can optimize only one goal (usually, the cost of feed rations), while the WGP method allows us to optimize numerous goals at the same time. LP and WGP also differ with respect to their target functions (TFs). The TF in LP is usually the cost of feed rations; in the WGP model, the TF is to minimize undesirable deviations from the target goal levels and not to minimize the goals themselves [1].

The aim of this paper is to present a user-friendly model for the optimization of feed rations for sport horses. Horse jumping is a sport that places physical and psychological burden on a horse ([3], [5]). Consequently, it is of utmost importance to provide optimal feed rations to satisfy all their daily requirements. Poorly balanced feed rations certainly affect the welfare of animals and can worsen their health conditions, especially in the case of animals which are under constant psychological and physical pressure, such as sport horses. Therefore, the rations must always be in line with the current needs of horses, which could change daily. Feeding of “sport” rations should start 8–12 weeks before intensive training activity [9], which means that unbalanced rations could have disastrous consequences on the gastrointestinal system, for example, development of gastric ulcers. Also, the rations need to be cost-effective.

Based on our literature review [5], [13], [14], we have developed a model that is based on two sub-models, LP and WGP.

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## METHODS

The traditional LP was used to calculate the cheapest rations, and the WGP method was used to balanced considering the minimum costs calculated with the LP model. The input data are the same for both sub-models, except that the calculated costs from LP were added to WGP input data (Figure 1). The input data include the daily requirements of horses with regard to nutrients and minerals: the amount of dry matter (DM), metabolisable energy (ME), metabolisable protein (MP), and calcium and phosphorus (the two most common minerals). For optimization, it is important to include the nutritive value of feed at disposal and their retail prices. In this paper, the rations were calculated using three different methods: in one, only LP was used, and in the other two, two different models of WGP, $WGP_{sc1}$ and $WGP_{sc2}$, were used.

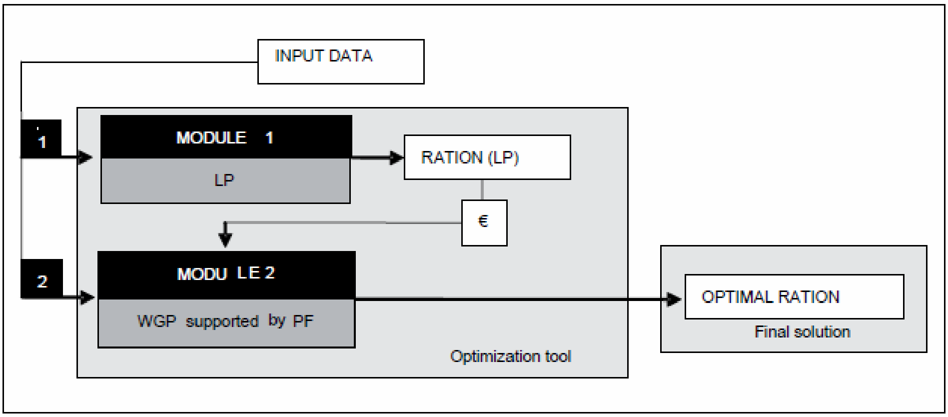


Figure 1 Optimization tool with two sub-models (LP and WGP) [13]

The restrictions in LP (represented by > or <) were transformed into goals represented by "=" in the WGP model. An important aim of the new model is to determine the positive and negative intervals in the WGP model. Intervals represent negative or positive deviations from daily nutrition requirements. Only in the case of DM, negative deviation was not defined, because horses show maximum consumption capacity for DM.

The next step was to define the weights (*w*) of separate goals. The "hanging" weights of goals of the decision maker have a hierarchical scale, with preference shown for certain goals. In the context of sport horses, it is important to satisfy all nutritive requirements within the available capacity of DM. The weights could have values between 0 and 100 or 0.0 and 1. With change in the weights, the decision maker could create numerous ration optimizations. During this survey, the weights of the costs changed ($WGP_{sc1}$ – w = 95 and $WGP_{sc2}$ – w = 5), which means that the cost of the rations used in $WGP_{sc1}$ should not deviate from the costs of ration which were calculated with LP. In $WGO_{sc2}$, the costs calculated from the LP model did not play a major role in the WGP optimization process. The input data from the paragraph above and this paragraph are represented in Table 1.

The role of TF in the WGP sub-model was not minimization of the costs of rations like in the LP sub-model, but determining the weighted sum of unwanted deviation of variables from observed goals, multiplied with the belonging coefficients for penalty ($pc_1$ = 2 and $pc_2$ =5). The obtained sum-product is the subject of minimization [13].

Table 1[2] Input data for the LP and WGP models (horses' daily nutrition requirements and defined penalty functions and weights)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **FEED STANDARDS** | | | | | | | |
| | **DM[1]** | **ME[1]** | **MP[1]** | **Ca[1]** | **P[1]** | | | |
| Weight of horse (550–600 kg) | **14** | **126,2** | **1207** | **36** | **26** | | | |
| Unit | kg/day | MJ/day | g/day | g/day | g/day | | | |
| **SPECIFIC INPUT DATA** | | | | | | | | |
| | | | | Penalty functions | | | | |
| | | | | Interval 1 | | Interval 2 | | Weights |
| | Unit | LP | $WGP_{sc1}$ and sc2 | $p_1^-$ | $p_1^+$ | $p_2^-$ | $p_2^+$ | $W_{sc1}(W_{sc2})$ |
| DM | (kg) | **<14** | **14** | **5%** | **0%** | **10%** | **0%** | **70 (70)** |
| ME | (MJ) | **>126,2** | **126,2** | **1%** | **3%** | **5%** | **10%** | **90 (90)** |
| MP | (g) | **>1207** | **1207** | **1%** | **3%** | **5%** | **10%** | **90 (90)** |
| Ca | (g) | **>37** | **37** | **10%** | **10%** | **20%** | **20%** | **5 (5)** |
| P | (g) | **>26** | **26** | **10%** | **10%** | **20%** | **20%** | **5 (5)** |
| Cost | (€) | | **C** | **∞** | **10%** | **∞** | **20%** | **5 (95)** |

The model was developed using the MS Office Excel 2010 framework tool. One part of the model is represented in Figure 2. It also depicts the second sub-model for the WGP mathematical technique with daily nutrition requirements, feed at disposal, defined goals, and penalty functions.

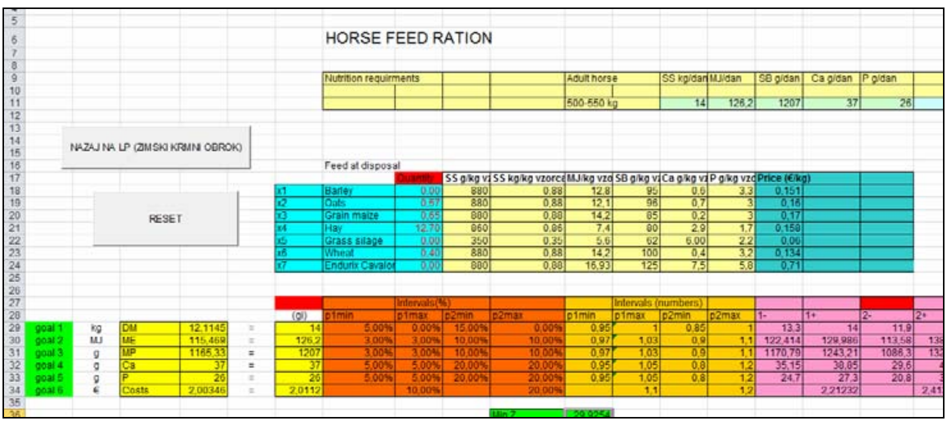


Figure 2 A part of the optimization model–the WGP sub-model

## RESULTS AND DISCUSSIONS

Results of the model are presented as calculated rations which are logical and can be used in practice. Rations are calculated up to two decimals for each feed, but in practice, the values could be rounded up to one decimal. Rations calculated using WGP contain more roughage, with 88% DM in the form of hay. In Table 2, three rations are presented (one LP-based and two WGP-

[2]The input data stand from feed standards normative for 550 to 600 kg horses which are included into active sport activities. Specific input data were defined for WGP sub-model and focused on definition of penalty intervals and weights of different goals.

based rations). In $WGP_{sc1}$, the cost of the rations was taken from the LP model (2,01€), but it is more balanced. There is no surplus of ME, MP, calcium, or phosphorus, which have an influence on animal health. According to Žgajnar et al. [13], this could indicate a conflict between nutrition quality and economics.

However, the results show an inverse relation between the content of dry matter containing hay and cost. Rations with hay include more dry matter compared with rations containing grass silage, but the former is more expensive. With the application of WGP, we are able to show that more balanced rations are not necessarily expensive at the same time (this was confirmed with $WGP_{sc2}$).

From a nutrition quality aspect, it could be concluded that rations calculated with WGP contained no surpluses and were more balanced than rations calculated with LP. The cost of the rations calculated with $WGP_{sc2}$ was 10% cheaper than that calculated with LP and $WGP_{sc1}$. The only possible problem is deviation from the DM normative. The negative deviation (-21.5%) could be compensated for with "blind" feed such as straw. Therefore, the only limitation that needs to be overcome is that the DM normative should be satisfied without any impact on other nutritive requirements.

Table 2 Calculated daily feed rations with LP and WGP for two scenarios

| Daily ration | | | | |
|---|---|---|---|---|
| Type of feed | Unit | LP | $WGP_{sc1}$ | $WGP_{sc2}$ |
| Hay | kg | **6.50** | **12.70** | **10.46** |
| Grass silage | kg | **3.50** | **0.00** | **1.05** |
| Grain maize | kg | **1.30** | **0.65** | **0.84** |
| Barley | kg | **0.70** | **0.00** | **0.00** |
| Oats | kg | **1.75** | **0.57** | **0.74** |
| Wheat | kg | **1.25** | **0.40** | **0.53** |
| Endurix Cavalor | kg | **0.00** | **0.00** | **0.00** |
| Cost | €/day | **2.01** | **2.01** | **1.79** |
| Cost deviation | % | | **0** | **-10.98** |
| Deviation from daily requirements | | | | |
| DM | **%** | **-19.89** | **-13.47** | **-21.50** |
| ME | **%** | **+6.22** | **-8.50** | **-11.73** |
| MP | **%** | **0.00** | **-3.45** | **-9.13** |
| Ca | **%** | **+14.20** | **0.00** | **0.00** |
| P | **%** | **+31.58** | **0.00** | **0.00** |

## CONCLUSIONS

The combination of LP and WGP described here can be directly used in practice. Moreover, rather than LP, an upgraded WGP method could be used to optimize several parameters at the same time, which could effectively help determine a more balanced ration. With the modified WGP optimization program, decision makers do not have to limit the convex function with small intervals and high goal weights, but they can create a logical hierarchical tree with preferred nutrients. In addition to its ability to adapt to changing needs for horses, this new method is also useful from the economic point of view, as it determines the cheapest rations with no surpluses.

## REFERENCES

[1] Ferguson E.L., Darmon N., Fhmida U., Fitriyanti S., Harper T.B., Premachandra I.M., 2006. Design of optimal food-based complementary feeding recommendations and identification of key "Problem Nutrients" using goal programming. J. Nutr. 136 (9), p. 2399-2404

[2] Gómez-Limón J.A., Riesgo L., Arriaza M., 2004. Multi-criteria analysis of input use in agriculture. J. Agric. Econ. 55, p. 541-564

[3] Janžekovič M., Prišenk, J., Muršec B., Vindiš P., Stajnko D., Čuš F., 2010. The art equipment for measuring the horse`s heart rate. J Achiev. Mater. Manuf. Eng. 41(1-2), p. 180-186.

[4] Lara P., 1993. Multiple objective fractional programming and livestock ration formulation: A case study for dairy cow diets in Spain. Agric. Syst. 41, p. 321-334
[5] Prišenk J., 2010. Heart rate monitoring by non-invasive method in sport horses and development of the linear model for calculation of the horse fodder. B.Sc. Thesis. University of Maribor, Faculty of Agriculture and Life Sciences, p. 64.
[6] Rehman T., Romero C., 1984. Multiple-criteria decision making techniques and their role in livestock ration formulation. Agric. Syst. 15, 23-49
[7] Rehman T., Romero C., 1987. Goal programming with penalty functions and livestock ration formulation. Agric. Syst. 23, p. 117-132
[8] Romero C., Rehman T., 2003. Multiple criteria analysis for agricultural decisions. 2nd ed. Elsevir, Amsterdam
[9] Salaić J., Baban M., Domaćinović M., Mijić P., Sakač M., Bobič T., Budimir K., 2010. Usporedba nutritivnih vrijednosti različitih obroka sportskih konja u zaprežnom sportu. Stoč. 64 (1), p. 29-39
[10] Stokes J.R., Tozer P.R., 2007. Cost minimization and managing soil nutrient loading: Conflict or compromise? Canad. J. Agric. Econ. 50 (2), p. 151-169
[11] Waugh F.V., 1951. The minimum-cost dairy feed. Am. J. Agric. Econ. 33, p. 299-310
[12] Zimmermann A., 2008. Optimization of sustainable dairy-cow feeding systems with an economic-ecological LP farm model using various optimization processes. J. Sust. Agric. 32 (1), p. 77-94
[13] Žgajnar J., Erjavec E., Kavčič S., 2010. Multi-step beef ration optimisation: application of linear and weighted goal programming with a penalty function. Agric. and food Sci. 19, p. 193-206
[14] Žgajnar J., Juvančič L., Kavčič S., 2009. Combination of linear and weighted goal programming with penalty function in optimisation of a daily dairy cow ration. Agric. Econ. 55, p. 492-500.