CONTRIBUTIONS TO THE DETERMINATION OF THE EQUATION FOR THE EQUIVALENT HEAD CHARACTERISTIC OF A HYDRAULIC GENERATOR BATTERY

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Abstract

Pumping stations that serve the pressurized and/or free level hydrotechnical systems are equipped, mainly, with several pumping sets, the turbopump (TP) – electric motor. TP have the role of hydraulic generators and present, in general, a parallel connection within the core of basic technological lines (LTB) of the pumping station. In the composition of a LTB there are suction and discharge lines of the TP, and the discharge pipeline (for example, a discharge collector). In order to satisfy in optimal conditions the variable flow requirements and pressure of the hydrotechnic system, a LTB can be equipped with TP of the same type or different types, and some TPs can be adjustable (with the positions of the stator blades and/or rotor blades that are variable, with the position given by the the angles s and/or r) or adjustable (at the variable turation n). The energetic active TP assembly on a LTB, together with the communication of suction and discharge, assemble a battery of hydraulic generators (BGH); the notion of BGH permits us the equalization, from a functional point of view of the respective assembly with an unique TP. The equivalent charge characteristic of the BGH (CSEBGH) relays the dependence between the pumping load relative to the discharge collector section, H_{BGH} and the total flow rate delivered to the discharge pipeline, Q_{BGH}. CSEBGH is useful in the analysis of the cooperation of BGH with the pipeline network of the served hydrotechnic system – the absolutely necessary analysis in the automation of the pumping stations through the SCADA system. In specialty literaturem the analytic expression for CSEBGH was determined only through its explicit form, usual for CS of a TP, HBGH=fH(QBGH), a form which is fossible only for some particular cases of BGH compoistion. In the current paper, for CSEBGH it is proposed a new analytic expression, trough the form forma QBGH=fQ(HBGH), which, by being available for every scenario of the composition of BGH, it presents the generality advantage. Also, for both forms of analytical expression exposed abovem there are presented representative numerical examples

Key words: turbopump, technological line base, hydraulic generators battery, characteristic equation equivalent

Pumping stations which serve under pressure and / or free-flow hydro technical systems are mainly equipped with several turbo pump (TP) pump units - an electric motor. TPs have the role of hydraulic generators and generally have a parallel coupling within the baseline technological lines (LTBs) of the pumping stations. In addition, a LTB (Alexandrescu O.,2004) is the suction and discharge communication of the TP, as well as the organs for connecting them to the discharge pipe line (for example- discharge collector). In order to meet the optimal flow and pressure requirements of the hydro technical system as well as for maintenance considerations, an LTB must be equipped with $1 \leq N_{tip} \leq 2$ TP types and some TPs can be adjusted (with the position of variable stator and / or rotor blades, position specified through the angles *s* and / or *r*) or driven by motors at variable speed *n.* Thus, in the most specific case, $N_{tip} = 2$, LTB is equipped with $m_P^I = m_{P0}^I + m_{\Pr}^I \geq 1$ type I TP, of which $m_{P0}^I \geq 0$ the TPs are normally unregulated and $m_{\Pr}^I \geq 0$ are adjustable at variable speed csf) as well as $m_P^{II} = m_{P0}^{II} + m_{\Pr}^{II} \geq 0$ Type II TP, of which $m_{P0}^{II} \geq 0$ the TPs are unregulated and $m_{\Pr}^{II} \geq 0$ are operated at variable speed.

The energy-efficient TP assembly on a LTB, together with its suction and discharge communications (hereinafter referred to as individual communications), comprise a battery of hydraulic generators (BGH); the notion of BGH (Alexandrescu O.,2004, Popescu Şt., 1992) allows us to functionally equate that assembly with a unique TP. Any scenario σ of configuration and operation of BGH can be characterized by the control vector (Popescu Şt., 1997, Chiorescu E.,

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2016) $\mathbf{u}_\sigma = \left\{\mathbf{u}_{\sigma 0}^{I}, \mathbf{u}_{\sigma r}^{I}, \mathbf{u}_{\sigma 0}^{II}, \mathbf{u}_{\sigma r}^{I}\right\}$, defined as follows next equations (1),(2):

$$\mathbf{u}_{\sigma 0}^{j} = \begin{cases} \Phi - \text{for } \left(m_{P0}^{j}\right)_\sigma = 0 \\ \left\{\left(m_{P0}^{j}\right)_\sigma, s_0^j, r_0^j, n_{\sigma 0}^{j}\right\} - \text{for } \left(m_{P0}^{j}\right)_\sigma > 0 \end{cases}, \text{with } j \in \{I, II\} \tag{1}$$

$$\mathbf{u}_{\sigma r}^{j} = \begin{cases} \Phi - \text{for } \left(m_{\mathrm{Pr}}^{j}\right)_\sigma = 0 \\ \left\{\left(m_{\mathrm{Pr}}^{j}\right)_\sigma, \left[\left(s_\sigma^j \text{ and/or } r_\sigma^j\right) \text{ or } n_\sigma^j\right]\right\} - \text{for } \left(m_{\mathrm{Pr}}^{j}\right)_\sigma > 0 \end{cases}, \text{with } j \in \{I, II\} \tag{2}$$

where:

Φ is wide multiple;

$0 \le \left(m_{P0}^{j}\right)_\sigma \le m_{P0}^{j}$ - the number of unregulated TPs of *j* type actually in operation;

$0 \le \left(m_{\mathrm{Pr}}^{j}\right)_\sigma \le m_{\mathrm{Pr}}^{j}$ - the number of adjustable TPs of *j* type actually in operation;

s_0^j - the constant positioning angle of the stator blades for non-adjustable TP j type;

r_0^j - the constant positioning angle of rotor blades for non-adjustable TP *j* type;

$n_{\sigma 0}^{j}$ - quasi-constant speed of asynchronous motor drive of non-adjustable TP *j* type ($n_{\sigma 0}^{j}$ can be rated according to the TP (Popescu Şt., 2003, Popescu Şt., 1999) or approximate with the nominal engine speed, n_0^j, $n_{\sigma 0}^{j} \cong n_0^j$);

s_σ^j -adjustable angle positioning of adjustable TP stator blades of *j* type;

r_σ^j - variable angle positioning of adjustable TP rotors for *j* type;

n_σ^j - the variable speed (via csf) of the TP *j* type.

Because the local load losses are predominant for individual communications, with the aim of simplifying exposure, we will still consider that the configurations of these communications are identical for all TPs of the same type; so we can assign the following equivalent hydraulic resistance modules: $M_{rp}^{j}, \text{with } j \in \{I, II\}$ - for all those m_P^j individual communications of ale TP *j* type, $\text{with } j \in \{I, II\}$.

The equivalent charge load characteristic of BGH (CSEBGH) offers the difference between the pumping load relative to a remarkable discharge section (HGH discharge section), HBGH, and the total flow rate delivered to the QBGH discharge pipe. In the scenario σ, CSEBGH is presented analytically by an explicit function in relation (3) to the Q_{BGH} flow,

$$Q_{\mathrm{BGH}} = f_Q\left(H_{\mathrm{BGH}}, \mathbf{u}_\sigma\right) \tag{3}$$

and only in some particular cases for the vector $\mathbf{u}_\sigma$ equation (3) can be explicit in relation to the task H_{BGH},

$$H_{\mathrm{BGH}} = f_H\left(Q_{\mathrm{BGH}}, \mathbf{u}_\sigma\right) \tag{4}$$

The determination of the CSEBGH is based on the following elements: 1- load characteristics of the energetically active TP (CS for the control vector $\mathbf{u}_{\sigma 0}^{j}$ - in the case of non-adjustable TPs driven at quasi-constant speed and / or control vector CS $\mathbf{u}_{\sigma 0}^{j}$ - in the case of TP adjustable and / or operated at variable speed); 2 - the characteristics of the load losses on the individual TP communications and 3 - the equation of continuity in the remarkable section of the discharge.

1. CS of a typical type TP is given in explicit form with respect to the pumping load, as follows relation (5), (6):

a) for an unregulated TP, driven at quasi-constant speed,

$$H_{pnr}^{j} = f_{Hnr}^{j}\left(Q_{pnr}^{j}, \mathbf{u}_{\sigma 0}^{j}\right), \text{with } \mathbf{u}_{\sigma 0}^{j} = \left\{1, s_0^j, r_0^j, n_{\sigma 0}^{j}\right\} \text{and } j \in \{I, II\} \tag{5}$$

b) for an unregulated TP, operated at variable speed,

$$H_{pr}^{j} = f_{Hr}^{j}\left(Q_{pr}^{j}, \mathbf{u}_{\sigma r}^{j}\right), \text{with } \mathbf{u}_{\sigma r}^{j} = \left\{1, s_0^j, r_0^j, n_\sigma^j\right\} \text{and } j \in \{I, II\} \tag{6}$$

In the two above-mentioned relations the following notations were introduced:

Q_{pnr}^{j}, Q_{pr}^{j} - the flow rate of TP of type j in cases a) and, respectively, b);

H_{pnr}^{j}, H_{pr}^{j} - the assignment of TP of type j in cases a) and, respectively, b).

2. The equation of the load loss characteristics on the individual J-type TP communications presents the following form (equation 7):

$$h_{rc}^{j} = h_{rc}^{j}\left(Q_p^j\right) = M_{rp}^{j} \cdot \left(Q_p^j\right)^2, \text{with } j \in \{I, II\} \tag{7}$$

where: h_{rc}^{j} is the loss of load on the individual J-type TP communications;

Q_p^j - the flow rate on the individual J-type TP communications, $Q_p^j \in \left\{Q_{pnr}^{j}, Q_{pr}^{j}\right\}$.

3. The equation of continuity in the remarkable section of the discharge shows the following general form (equation 8):

$$Q_{\mathrm{BGH}} = \sum_{j \in \{I, II\}} \left[\left(m_{P0}^{j}\right)_\sigma \cdot Q_{pnr}^{j} + \left(m_{\mathrm{Pr}}^{j}\right)_\sigma \cdot Q_{pr}^{j}\right] \tag{8}$$

The low TP load, in relation to the remarkable discharge section, is obtained by lowering the load losses of the pump on the individual TP communications; thus, from the equations (5) ... (7) we obtain the following low load characteristics (CSred):

a) CSred for an unregulated TP, driven at quasi-constant speed in (equation 9):,

$$\overline{H}^{j}_{pnr} = H^{j}_{pnr} - h^{j}_{rc}\left(Q^{j}_{pnr}\right) = f^{j}_{Hnr}\left(Q^{j}_{pnr}, \mathbf{u}^{j}_{\sigma 0}\right) - M^{j}_{rp}\cdot\left(Q^{j}_{pnr}\right)^{2}, \text{with } j \in \{I, II\} \quad (9)$$

b) CSred for an unregulated TP, operated at variable speed in (equation 10):,

$$\overline{H}^{j}_{pr} = H^{j}_{pr} - h^{j}_{rc}\left(Q^{j}_{pr}\right) = f^{j}_{Hr}\left(Q^{j}_{pr}, \mathbf{u}^{j}_{\sigma r}\right) - M^{j}_{rp}\cdot\left(Q^{j}_{pr}\right)^{2}, \text{with } j \in \{I, II\} \quad (10)$$

In the two above-mentioned relationships and represents the reduced load related to TP of type j in cases a) and b), respectively.

Assuming that the mass of water in the collector is at rest, from the application of the Pascal principle, the following equations result (equation 11):

$$\overline{H}^{j}_{pnr} = H_{\text{BGH}}, \text{with } j \in \{I, II\} \quad \text{and} \quad \overline{H}^{j}_{pr} = H_{\text{BGH}}, \text{with } j \in \{I, II\} \quad (11)$$

Substituting each of the equations (11) in the CSred equations (9) and (10), results (equations 12, 13):

$$f^{j}_{Hnr}\left(Q^{j}_{pnr}, \mathbf{u}^{j}_{\sigma 0}\right) - M^{j}_{rp}\cdot\left(Q^{j}_{pnr}\right)^{2} = H_{\text{BGH}}, \text{with } j \in \{I, II\} \quad (12)$$

$$f^{j}_{Hr}\left(Q^{j}_{pr}, \mathbf{u}^{j}_{\sigma r}\right) - M^{j}_{rp}\cdot\left(Q^{j}_{pr}\right)^{2} = H_{\text{BGH}}, \text{with } j \in \{I, II\} \quad (13)$$

Explaining from equations (12) and (13) respectively flows Q^{j}_{pnr}, with $j \in \{I, II\}$ and, respectively Q^{j}_{pr}, with $j \in \{I, II\}$, and then introducing them into the continuity equation (8), the CSEBGH equation (3) results.

The usual CSI analytical expressions of a non-adjustable, non-adjustable, TP type quasi-regular speed are as follows:

a) Complete form using equations: (1), (2), (6), (7), (8),

$$H^{j}_{pnr} = a^{j}_{H0} + b^{j}_{H0}\cdot Q^{j}_{pnr} + c^{j}_{H0}\cdot\left(Q^{j}_{pnr}\right)^{2}, \text{with } j \in \{I, II\} \quad (14)$$

b) reduced form equations: (3), (4), (5), (8) are:

$$H^{j}_{pnr} = H^{j}_{p0} - k^{j}_{p0}\cdot\left(Q^{j}_{pnr}\right)^{2}, \text{with } j \in \{I, II\} \quad (15)$$

By introducing equations (14) and (15) in equation (8), the following analytical expressions for CSred of an unregulated TP, driven at quasi-constant speed:

a) complete form,

$$\overline{H}^{j}_{pnr} = a^{j}_{H0} + b^{j}_{H0}\cdot Q^{j}_{pnr} + \overline{c}^{I}_{H0}\cdot\left(Q^{j}_{pnr}\right)^{2}, \text{with } j \in \{I, II\} \quad (16)$$

b) reduced form,

$$\overline{H}^{j}_{pnr} = H^{j}_{p0} - \overline{k}^{j}_{p0}\cdot\left(Q^{j}_{pnr}\right)^{2}, \text{with } j \in \{I, II\} \quad (17)$$

where the notations have been entered below

$$\overline{c}^{I}_{H0} = c^{j}_{H0} - M^{j}_{rp}, \ \overline{k}^{j}_{p0} = k^{j}_{p0} + M^{j}_{rp} \quad (18)$$

By introducing conditions (11) into equations (14) and (15), the Q^{j}_{pnr} flow can be explained as follows:

a) for the complete form in equation (19):

$$Q^{j}_{pnr} = \frac{-b^{j}_{H0} + \sqrt{\left(b^{j}_{H0}\right)^{2} - 4\cdot\overline{c}^{j}_{H0}\cdot\left(a^{j}_{H0} - H_{\text{BGH}}\right)}}{2\cdot\overline{c}^{j}_{H0}}, \text{with } j \in \{I, II\} \quad (19)$$

With notations

$$\overline{A}^{j}_{H0} = -\frac{b^{j}_{H0}}{2\overline{c}^{j}_{H0}}; \overline{B}^{j}_{H0} = \left(\overline{A}^{j}_{H0}\right)^{2} - \frac{a^{j}_{H0}}{\overline{c}^{j}_{H0}}, \text{with } j \in \{I, II\} \quad (20)$$

ecuation (19) becomes:

$$Q^{j}_{pnr} = \overline{A}^{j}_{H0} + \sqrt{\overline{B}^{j}_{H0} + \left(1/\overline{c}^{j}_{H0}\right)H_{\text{BGH}}}, \text{with } j \in \{I, II\} \quad (21)$$

a) reduced form,

$$Q^{j}_{pnr} = \sqrt{\frac{H^{j}_{p0} - H_{\text{BGH}}}{\overline{k}^{j}_{p0}}}, \text{with } j \in \{I, II\} \quad (22)$$

By introducing the expressions (21) or (22) into the continuity equation (8), the following equations result for CSEBGH in form (3):

a) for the complete form

$$Q_{\text{BGH}} = \sum_{j\in\{I,II\}}\left(m^{j}_{P0}\right)_{\sigma}\left[\overline{A}^{j}_{H0} + \sqrt{\overline{B}^{j}_{H0} + \left(1/\overline{c}^{j}_{H0}\right)H_{\text{BGH}}}\right] \quad (23)$$

a) for the reduced form

$$Q_{\text{BGH}} = \sum_{j\in\{I,II\}}\left(m^{j}_{P0}\right)_{\sigma}\cdot\sqrt{\frac{H^{j}_{p0} - H_{\text{BGH}}}{\overline{k}^{j}_{p0}}} \quad (24)$$

In the specialized literature CSEBGH (23) and (24) were explained in relation to the HBGH load, in form (4) for two particular scenarios, related to the following expressions for the control vector $\mathbf{u}_{\sigma} = \left\{\mathbf{u}^{I}_{\sigma 0}, \mathbf{u}^{I}_{\sigma r}, \mathbf{u}^{II}_{\sigma 0}, \mathbf{u}^{I}_{\sigma r}\right\}$:

$$1°\text{-}\mathbf{u}_{\sigma} = \left\{\left\{1 \le \left(m^{I}_{P0}\right)_{\sigma} \le m^{I}_{P0}, s^{I}_{0}, r^{I}_{0}, n^{I}_{\sigma 0}\right\}, \Phi, \Phi, \Phi\right\}$$

a. complete form, with 3 coefficients (6),(7), (8),

$$H_{\text{BGH}} = a^{I}_{H0} + \frac{b^{I}_{H0}}{\left(m^{I}_{P0}\right)_{\sigma}}\cdot Q_{\text{BGH}} + \frac{\overline{c}^{I}_{H0}}{\left[\left(m^{I}_{P0}\right)_{\sigma}\right]^{2}}\cdot\left(Q_{\text{BGH}}\right)^{2} \quad (25)$$

a) Reduced form, cu 2 coefficients

$$H_{\mathrm{BGH}} = H_{p0}^{I} - \frac{\overline{k}_{p0}^{I}}{\left[\left(m_{P0}^{I}\right)_{\sigma}\right]^{2}} \cdot \left(Q_{\mathrm{BGH}}\right)^{2} \qquad (26)$$

2°

$$\mathbf{u}_{\sigma} = \left\{\left\{1 \le \left(m_{P0}^{I}\right)_{\sigma} \le m_{P0}^{I}, s_0^{I}, r_0^{I}, n_{\sigma 0}^{I}\right\}, \Phi, \left\{1 \le \left(m_{P0}^{II}\right)_{\sigma} \le m_{P0}^{II}, s_0^{II}, r_0^{II}, n_{\sigma 0}^{II}\right\}, \Phi\right\}$$

b.Complete form, cu 5 coefficients (6), (7), (8),

$$H_{\mathrm{BGH}} = b_{\mathrm{BGH}} + c_{\mathrm{BGH}} \cdot \left(Q_{\mathrm{BGH}} - a_{\mathrm{BGH}}\right)^{2} + d_{\mathrm{BGH}} \cdot \left(Q_{\mathrm{BGH}} - a_{\mathrm{BGH}}\right)\sqrt{\left(Q_{\mathrm{BGH}} - a_{\mathrm{BGH}}\right)^{2} + e_{\mathrm{BGH}}} \qquad (27)$$

where coefficients a_{BGH}, b_{BGH}, c_{BGH}, d_{BGH} si e_{BGH} are dependent only by the coefficients CSred (16) and the number of active TPs $\left(m_{P0}^{j}\right)_{\sigma}$, $0 \le \left(m_{P0}^{j}\right)_{\sigma} \le m_{P0}^{j}$, with $j \in \{I, II\}$.

<u>*b.Reduced form, with 4 coefficients*</u> (3),(4),(5),(8),

$$H_{\mathrm{BGH}} = a_{\mathrm{BGH}} + b_{\mathrm{BGH}} \cdot \left(Q_{\mathrm{BGH}}\right)^{2} + c_{\mathrm{BGH}} \cdot Q_{\mathrm{BGH}}\sqrt{\left(Q_{\mathrm{BGH}}\right)^{2} + d_{\mathrm{BGH}}} \qquad (28)$$

where coefficients a_{BGH}, b_{BGH}, c_{BGH} si d_{BGH} are dependent only by the coefficients CSred (17) and the number of active TPs $\left(m_{P0}^{j}\right)_{\sigma}$, $0 \le \left(m_{P0}^{j}\right)_{\sigma} \le m_{P0}^{j}$, with $j \in \{I, II\}$.

In the present paper, for CSEBGH is deduced a new analytical expression, explicit in relation to the QBGH flow, of the form (3), which, being valid for any scenario of BGH construction and operation, including control vectors, has a general advantage.

MATERIAL AND METHOD

Due to certain technical and economic advantages, the LTB of SP has now been extended with variable speed pumping units using CSF. Thus, without limiting the generality of the exposure but only for fixing the ideas, the control vectors $\mathbf{u}_{\sigma r}^{j} \ne \Phi$, with $j \in \{I, II\}$, through the relations below (29):

$$\mathbf{u}_{\sigma r}^{j} = \left\{\left(m_{\mathrm{Pr}}^{j}\right)_{\sigma}, s_0^{j}, r_0^{j}, n_{\sigma}^{j}\right\}, \text{ with } j \in \{I, II\} \qquad (29)$$

Deduction of the concrete analytical expression for CSEBGH in the general case is based on the continuity equation (8), in which the flows Q_{pnr}^{j}, with $j \in \{I, II\}$, are presented by the equations (21), whereas the flows Q_{pr}^{j}, with $j \in \{I, II\}$, must be extracted from the form equations (13). For this purpose, the CS of a TP of j type (6), formed all of the following equations (1), (2), (6), (7), (8) we have expression (30):

$$H_{pr}^{j} = f_{Hr}^{j}\left(Q_{pr}^{j}, \mathbf{u}_{\sigma r}^{j}\right) = a_{Hn}^{j} \cdot \left(n_{\sigma}^{j}\right)^{2} + b_{Hn}^{j} \cdot Q_{pr}^{j} \cdot n_{\sigma}^{j} + c_{Hn}^{j} \cdot \left(Q_{pr}^{j}\right)^{2}, \text{with } j \in \{I, II\} \qquad (30)$$

Introducing the equations above in equations (13), result the following:

$$a_{Hn}^{j} \cdot \left(n_{\sigma}^{j}\right)^{2} + b_{Hn}^{j} \cdot Q_{pr}^{j} \cdot n_{\sigma}^{j} + \overline{c}_{Hn}^{j} \cdot \left(Q_{pr}^{j}\right)^{2} = H_{\mathrm{BGH}}, \text{with } j \in \{I, II\} \qquad (31)$$

where

$$\overline{c}_{Hn}^{I} = c_{Hn}^{j} - M_{rp}^{j}, \text{with } j \in \{I, II\} \qquad (32)$$

Positive solutions of second degree equations (31) in unknowns Q_{pr}^{j}, with $j \in \{I, II\}$, are as follow (equation 33):

$$Q_{pr}^{j} = \overline{A}_{Hn}^{j} n_{\sigma}^{j} + \sqrt{\overline{B}_{Hn}^{j}\left(n_{\sigma}^{j}\right)^{2} + \left(1/\overline{c}_{Hn}^{j}\right) H_{\mathrm{BGH}}}, \text{with } j \in \{I, II\} \qquad (33)$$

where the notations were introduced

$$\overline{A}_{Hn}^{j} = -\frac{b_{Hn}^{j}}{2\overline{c}_{Hn}^{j}}; \overline{B}_{Hn}^{j} = \left(\overline{A}_{Hn}^{j}\right)^{2} - \frac{a_{Hn}^{j}}{\overline{c}_{Hn}^{j}}, \text{with } j \in \{I, II\} \qquad (34)$$

Introducing flows, Q_{pnr}^{j} and Q_{pr}^{j}, respectively, result the equations (21) and (33) in the continuity equation (8), CSEBGH is obtained under the following general form (35):

$$Q_{\mathrm{BGH}} = \sum_{j \in \{I, II\}} \left\{\left(m_{P0}^{j}\right)_{\sigma}\left[\overline{A}_{H0}^{j} + \sqrt{\overline{B}_{H0}^{j} + \left(1/\overline{c}_{H0}^{j}\right) H_{\mathrm{BGH}}}\right] + \left(m_{\mathrm{Pr}}^{j}\right)_{\sigma}\left[\overline{A}_{Hn}^{j} n_{\sigma}^{j} + \sqrt{\overline{B}_{Hn}^{j}\left(n_{\sigma}^{j}\right)^{2} + \left(1/\overline{c}_{Hn}^{j}\right) H_{\mathrm{BGH}}}\right]\right\}$$

$$\text{for } H_{\mathrm{BGH}} \in \left[0, H_{\mathrm{BGH}}^{\max}\right] \qquad (35)$$

The domain of definition of the CSEBGH equation (35) is deduced from the following inequalities (36):

$$\overline{B}_{H0}^{j} + \left(1/\overline{c}_{H0}^{j}\right) H_{\mathrm{BGH}} \ge 0; \overline{B}_{Hn}^{j}\left(n_{\sigma}^{j}\right)^{2} + \left(1/\overline{c}_{Hn}^{j}\right) H_{\mathrm{BGH}} \ge 0, \text{with } j \in \{I, II\} \qquad (36)$$

where the upper boundary of the domain results, $H_{\mathrm{BGH}}^{\max}$:

$$H_{BGH}^{\max} = \min_{j \in \{I, II\}} \left\{-\overline{c}_{H0}^{j} \cdot \overline{B}_{H0}^{j}, -\overline{c}_{Hn}^{j} \cdot \overline{B}_{Hn}^{j}\left(n_{\sigma}^{j}\right)^{2}\right\} \qquad (37)$$

RESULTS AND DISCUSSIONS

The theoretical considerations presented above were applied to LTB of SRPA 21 Viziru, Brăila County, which is equipped as follows:

I°- with m_P^I=3 TP type NDS 400-350-510 (considered as "large" TP), with rotor Φ 510, rated speed n_0^I=1450 rpm, each having nominal flow $Q_p^{nom\_I}$=1620 m.c./h; the inner communications of the 3 "big" TPs are identical and have the module M_{rp}^I=12.86 m^{-5}s^2;

m_{P0}^I=2 TP "Large" are irregularly driven at speed n_0^I, for which CS coefficients (14) are entered in Table 1, line 2, and m_{Pr}^I=1 TP are operated at variable speed by means of a CSF and for which the CS coefficients (16) are entered in Table 1, line 5;

II°- with m_P^{II} =2 TP type NDS 250-200-510 (considered as "small" TP), with rotor Φ 500, rated speed n_0^{II} =1450 rpm, each having the nominal flow Q_p^{nom-II} =540 m^3/h; the inner communications of the two "small" TPs are identical and have the module M_{rp}^I =126.50 $m^{-5}s^2$; m_{P0}^{II} =1 TP "Small" are acting irregularly at speed n_0^{II}, for which CS coefficients (14) are entered in Tab. 1, line 3, and, m_{Pr}^{II} =1 TP are operated at variable speed by means of a CSF and for which the CS coefficients (16) are entered in Table1, line 6.

The coefficients of the type CSEBGH (35), which were determined with equations (20) and (34), are centralized in the Table 2; with the values of these coefficients, equation (35) is customized accordingly like this (equation 38):

$$Q_{\mathrm{BGH}} = \left(m_{P0}^I\right)_\sigma \left(0.1723692 + \sqrt{0.5340560\text{-}0.005771 H_{\mathrm{BGH}}}\right) + \left(m_{P0}^{II}\right)_\sigma \left(0.0515783 + \sqrt{0.0551514\text{-}5.84770\mathrm{e}^{-4}}\right) +$$
$$\left(m_{\mathrm{Pr}}^I\right)_\sigma \left(1.1696\mathrm{e}^{-4} n_\sigma^j + \sqrt{2.5396\mathrm{e}^{-7}\left(n_\sigma^j\right)^2 \text{-}0.00576 H_{\mathrm{BGH}}}\right) + \left(m_{\mathrm{Pr}}^{II}\right)_\sigma \left(3.6052\mathrm{e}^{-5} n_\sigma^j + \sqrt{2.5378\mathrm{e}^{-8}\left(n_\sigma^j\right)^2 \text{-}5.652\mathrm{e}^{-4} H_{\mathrm{BGH}}}\right) \quad (38)$$

The following were considered to be 11 representative BGH operating scenarios characterized by the control vector (Popescu Şt., 1997, Chiorescu E., 2016).

$\mathbf{u}_\sigma = \left\{\mathbf{u}_{\sigma 0}^I, \mathbf{u}_{\sigma r}^I, \mathbf{u}_{\sigma 0}^{II}, \mathbf{u}_{\sigma r}^I\right\}$ in equation (39),

$$\mathbf{u}_{\sigma 0}^I = \left\{m_{\mathrm{P}0}^I\right\}, \mathbf{u}_{\sigma r}^I = \left\{m_{\mathrm{Pr}}^I, n_\sigma^I\right\}, \mathbf{u}_{\sigma 0}^{II} = \left\{m_{\mathrm{P}0}^{II}\right\}, \mathbf{u}_{\sigma r}^I = \left\{m_{\mathrm{Pr}}^{II}, n_\sigma^{II}\right\} \quad (39)$$

contained in the (Table 3 –columns 1…7), and the CSEBGH results, consisting of the values of the coefficients of the equation (35) and the upper limit of the domain, $H_{\mathrm{BGH}}^{\max}$, are centralized in Tab. 3- col. 3 ... 9.

The equations for CSEBGH related to the 11 scenarios were graphically represented in Figure1.

Table 1

CS coefficients (load characteristics) for TP (turbo pumps) in equipment The LTB (base line) of the SP (pumping station)

	Speed	Coefficients		1	2	3
1	Constant, $n_{\sigma 0}^j$	Symbol		a_{H0}^j	b_{H0}^j	c_{H0}^j
2		Values	*j*=I: NDS 400-350-510	87.397590	59.739493	-160.42935
3			*j*=II: NDS 250-200-510	89.763537	176.40538	-1583,57421
4	Variable, $n_\sigma^j < n_{\sigma 0}^j$	Symbol		a_{Hn}^j	b_{Hn}^j	c_{Hn}^j
5		Values	*j*=I: NDS 400-350-510	4.157373e-5	0.0406335	-160.844010
6			*j*=II: NDS 250-200-510	4,260169e-5	0.1276722	-1642.80376

Table 2

Coefficients of the CSEBGH equation

	Speed	Coefficients		1	2	3
1	Constant, $n_{\sigma 0}^j$	Symbol		$\bar{A}_{H0}^j$	$\bar{B}_{H0}^j$	$1/\bar{c}_{H0}^j$
2		Values	*j*=I: NDS 400-350-510	0.1723692	0.5340560	-0.005771
3			*j*=II: NDS 250-200-510	0.0515783	0.0551514	-5.84770e-4
4	Variable, $n_\sigma^j < n_{\sigma 0}^j$	Symbol		$\bar{A}_{Hn}^j$	$\bar{B}_{Hn}^j$	$1/\bar{c}_{Hn}^j$
5		Values	*j*=I: NDS 400-350-510	1.169619e-4	2.539584e-7	-0.0057569
6			*j*=II: NDS 250-200-510	3.6051551-5	2.537794e-8	-5.65194e-4

Table 3

The elements of control vector $\mathbf{u}_\sigma$ and particular equations for CSEBGH in 11 scenarios of BGH operation.

σ	$\mathbf{u}_\sigma$						Equation for CSEBGH as $Q_{\mathrm{BGH}} = f_Q\left(H_{\mathrm{BGH}}, \mathbf{u}_\sigma\right)$	$H_{\mathrm{BGH}}^{\max}$
	$m_{\mathrm{P}0}^I$	m_{Pr}^I	n_σ^I	$m_{\mathrm{P}0}^{II}$	m_{Pr}^{II}	n_σ^{II}		
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	0	0	0	0	1	1197	$0.0431496 + \sqrt{0.0363549\text{ -}5.651941\mathrm{e}^{-4} H_{\mathrm{BGH}}}$	64.0
2	0	0	0	1	0	0	$0.0515783 + \sqrt{0.0551514\text{-}5.847699\mathrm{e}^{-4} H_{\mathrm{BGH}}}$	94.3
3	0	1	1261	0	0	0	$0.1474329 + \sqrt{0.4035176\text{-}0.0057569 H_{\mathrm{BGH}}}$	70.0

4	1	0	0	0	0	0	$0.1723692+\sqrt{0.5340560\text{-}0.005771H_{\mathrm{BGH}}}$	92.5
5	0	0	0	2	0	0	$2\cdot\left(0.0515783+\sqrt{0.0551514\text{-}5.847699e^{-4}H_{\mathrm{BGH}}}\right)$	94.3
6	2	0	0	0	0	0	$2\cdot\left(0.1723692+\sqrt{0.5340560\text{-}0.005771H_{\mathrm{BGH}}}\right)$	92.5
7	0	0	0	1	1	1197	$0.09473+\sqrt{0.05515\text{-}5.8477e^{-4}H_{\mathrm{BGH}}}+\sqrt{0.03635\ \text{-}5.6519e^{-4}H_{\mathrm{BGH}}}$	64.0
8	1	0	0	0	1	1197	$0.21552+\sqrt{0.53406\text{-}0.00577H_{\mathrm{BGH}}}+\sqrt{0.03635\ \text{-}5.6519e^{-4}H_{\mathrm{BGH}}}$	64.0
9	1	1	1261	0	0	0	$0.3198+\sqrt{0.53406\text{-}0.00577H_{\mathrm{BGH}}}+\sqrt{0.40352\text{-}0.00576H_{\mathrm{BGH}}}$	70.0
10	1	0	0	1	0	0	$0.22395+\sqrt{0.53406\text{-}0.00577H_{\mathrm{BGH}}}+\sqrt{0.05515\text{-}5.8477e^{-4}H_{\mathrm{BGH}}}$	92.5
11	2	0	0	2	0	0	$2\left(0.22395+\sqrt{0.53406\text{-}0.00577H_{\mathrm{BGH}}}+\sqrt{0.05515\text{-}5.8477e^{-4}H_{\mathrm{BGH}}}\right)$	92.5

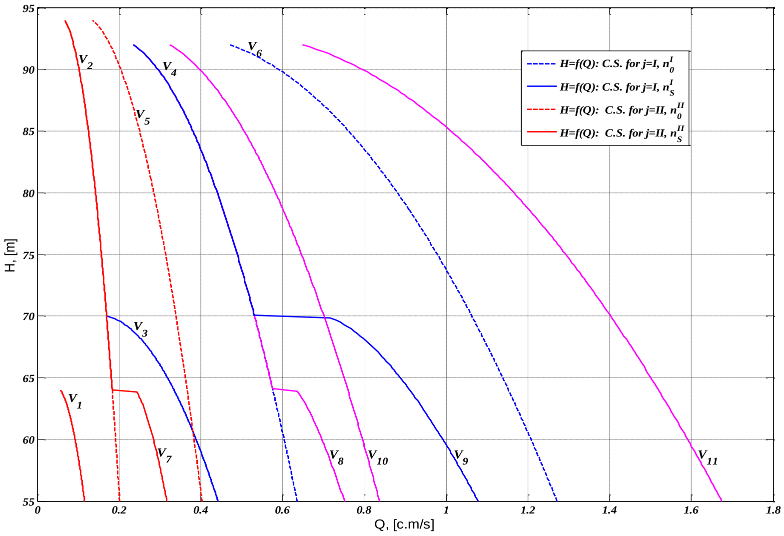

Figure 1 **Graphical representation of CSEBGH in 11 scenarios of BGH operation.**

CONCLUSIONS

There is a representative documentary synthesis on the current mathematical modeling of CSEBGH, which shows that only BGHs have been treated so far that include only unregulated TPs, driven at quasi-wind speed.

In the present paper the mathematical modeling of CSEBGH is approached within the theory of systems, which led to the deduction of a general analytical expression, valid for any scenario of BGH construction and functioning and / or driven by adjustable speed motors.

Since the LTB of SP has been expanded with variable speed pumping units using csf, the Material and method section has customized the CSEBGH general equation in this scenario of BGH construction and operation.

Theoretical results mentioned at the point were applied for a representative Case Study.

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