THE ACOUSTIC CHARACTERISATION OF CHUA’S CHAOS CIRCUIT

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Abstract

Chua’s circuit has multiple applications - in politics, economics, telecommunications, technical physics, nanomaterial synthesis, biology - and is of interest thanks to the multitude of adjustment parameters in simulation processes. The paper presents the detailed acoustic analysis which represents a benchmark in relation to other technical systems, for example, DBD plasma.

Keywords: Chua’s circuit, bifurcation, Matlab functions, sonogram, frequency, output

General overview of Chua’s circuit

In the world of artistic entertainment, stage lighting based on the dynamic bifurcation of the spotlight is used, cf. figure 1 (Murali K. and Lakshmanan M., 1990).

To achieve the scene effect (Leon O. Chua), 1992, the control electronic circuit includes the Chua circuit (Leon O. Chua; Kılıç Recai, 2010) consisting of five linear components - inductance L with internal resistance R, two capacitors C1 and C2, an external resistor R and a nonlinear component - an NR memristor or Cua diode as negative resistance, figure 2.

MATERIAL AND METHOD

The dynamic electrical system with three degrees of freedom is described by the system of equations below:

\[
\begin{align*}
\frac{dx}{dt} &= K\alpha[y - x - f(x)] \\
\frac{dy}{dt} &= K\alpha(x - y + z) \\
\frac{dz}{dt} &= k(-f y - \gamma)z \\
f(x) &= \frac{1}{2}(a - b) \cdot \{ |x + 1| - |x - 1| \} 
\end{align*}
\]

where \(y, \beta, a, b, k\) are adjusting variables. The fine change in the parameters yields numerous forms of three-dimensional fractal structures that may be used to synthesize musical compositions.

For example, for a model, the control function takes the following form:

\[
R = 1300; \\
R1 = 1200; \\
R2 = 3300; \\
r = 85; \\
C1 = 4.28e-9; \\
C2 = 69e-9; \\
L = .0085; \\
a = R1./R; \\
b = 1-R1./R2; \\
c = (C1.*(R1)^2)./L; \\
s = C1./C2; \\
p = r./R1; \\
F = @(t,y) [a*(y(2,:)-y(1,:))-gbar(y(1,:),b); s*(-a*(y(2,:)-y(1,:)))+y(3,:); -c*(y(2,:)+p*y(3,:))];
\]

which can be generated up with the Matlab function ode45:

\[
[tS, yS] = ode45(F, [0 N], yo, odeset ('reltol', 1e-6, 'abstol', 1e-9));
\]
by setting the initial conditions:
N = 10000;
y0 = [.1;.15;.05];
The following values result:
x1 = (1-b)/((a/(1+a*p))-b);
y1 = a*p*x1/(1+a*p);
z1 = -y1/p;
which can be analysed and represented graphically, Figure 1 sq.

RESULTS AND DISCUSSIONS

Experimental part. Work methodology
The acoustic signal of the .avi format video resulting from the analysis of oscillograms was transferred to an .mp3 format file and then converted to .wav format with the following characteristics:
<table>
<thead>
<tr>
<th>Length</th>
<th>06:01:07</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audio</td>
<td></td>
</tr>
<tr>
<td>Bit rate</td>
<td>512kbps</td>
</tr>
<tr>
<td>Name</td>
<td>Chua film.wav</td>
</tr>
<tr>
<td>Item type</td>
<td>Wave Sound</td>
</tr>
</tbody>
</table>

that can be processed using the MATLAB function:
[y, f, nbits] = wavread('Chua film.wav'),

Preliminary findings
Consecutive values in the y column vector y (single channel recording) were arranged in the three-dimensional vector tsd1 which, in the 3D representation, renders the spatial dispersion, figure 3.

tsd1 = y(3:end,1) y(2:end-1,1) y(1:end-2,1); figure; plot3(tsd1(:,1), tsd1(:,2), tsd1(:,3), ',', 'markersize', 1);
The representations in figure 2 indicate a spatial body with uneven distribution of consecutive values and an emerging state of chaos, correlated with stage lighting.

**Acoustic analysis**

During the recording:

t=size(y)./f, in seconds the change in acoustic signal amplitude was emphasised, figure 4.

By employing the Fourier transform as follows:

\[ p = \text{fft}(y(:,1)); \]

the diagram in figure 6 is generated.

![Power-frequency spectrogram](image)

**Figure 6** Power-frequency spectrogram

Using the sequence:

\[
\begin{align*}
X &= \text{fft}(y); \\
n &= \text{length}(y); \\
P &= X .* \text{conj}(X) / n; \\
w &= f * (0:(n/2 - 1))/n; \\
\end{align*}
\]

results the magnitude of absolute values

\[ y_{\text{fft}} = \text{abs}(	ext{fft}(y)); \]

from function

\[ y_{\text{fft}} = y_{\text{fft}}(1: \text{Nsamps}/2); \]

presented in Figure 7 with the characteristic values:

\[
\begin{align*}
\text{rms}_\text{val} &= 0.0871; \\
\text{Et} &= 121.5017 \text{ W}\n\end{align*}
\]

and the amplitude cumulated by frequencies, figure 8.

![Acoustic power - frequency](image)

**Figure 7** Acoustic power - frequency

![Cumulated amplitude - frequency](image)

**Figure 8** Cumulated amplitude - frequency

In a limited frequency field, the representation in figure 9 is obtained.

![Maximum range of acoustic power](image)

**Figure 9** Detail in figure 6

On the logarithmic scale the peak in maximum power is achieved at 1400 Hz frequency, figure 10.

The spectrogram developed using the function:

\[ \text{specgram}(y, 512, f); \]

indicates the uneven scattering at different frequencies in time, figure 11.

![Spectrogram](image)

**Figure 11** Spectrogram

h = spectrum.welch;  
spectral estimator.  
Hpsd = psd(h,y,'fs',f);  
figure  
plot(Hpsd)

![Distribution of spectral power](image)

**Figure 12** Distribution of spectral power
CONCLUSIONS

Compared to the existing state of technology - the electrotechnical representation of oscillograms - the paper presents a detailed acoustic analysis of the Chua dynamic circuit.

Data analysis yields the following results:
- change in amplitude over time, figures 4, 5 and 9;
- acoustic power (W) in frequency dependency with two maximum values, figure 7, and total energy $E_t = 121.5017$ W;
- broad frequency spectrum in the 0 - 2000 Hz range, cf. detail in figure 9;
- the maximum peak of acoustic power (dB) achieved at the 1400 Hz frequency, figure 10;
- unevenness of the spectrogram, figure 11;
- increase in specific acoustic power (noise) (dB/Hz) along with the increase in frequency.

REFERENCES

Leon O. Chua, 1992, Chaos synchronization in Chua's circuit, Berkeley: Electronics Research Laboratory, College of Engineering, University of California, OCLC: 44107698;
Dao Tran, 2014, Chaotic systems and Chua's Circuit, KME Regional Meeting, Emporia State University, KANSAS.

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https://www.youtube.com/watch?v=WRXP0ZetFlM.