CONTRIBUTIONS TO THE HYDRAULIC DETERMINATION OF DRAINAGE SYSTEMS WITH JUNCTION CHAMBERS

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Abstract

Drains and collectors have the role of collecting the water from an aquifer as well as transporting it to the collecting chamber. In transit, the drains are equipped with inspection chambers at no more than 400 m from each other that serve not only for the maintenance but also as hydraulic junctions for successive sections when there are changes of slope, direction or size, in which case the location distance may be much smaller (Arsenie M., 1982). For the drainage systems (Bârsan E., 2001,), from the hydraulic point of view, the following conceptual models are considered: 1^o closed drain with cylindrical/prismatic bed, unconfined, in which there is gradually changing motion and the flow rate varies - on the sections between two successive chambers; 2^o singularity with sudden change of section, with rapidly varied movement regime - on crossing a junction chamber. While for the first conceptual model there are already established, real life tested mathematical models, for the second one there are no satisfactory theoretical approaches. Starting from the general equations of hydrodynamics, like the continuity equation, we designed in this paper a mathematical model to determine the functional relation between water levels from the two ends of the two sections of the drain connected by a chamber. This model was implemented in a complex computer program of flow simulation for the entire length of the drainage system, and used to solve a representative study case.

Key words: drainage system, junction chamber, rapidly varied motion, mathematical modeling, numerical simulation

The mathematical models for the auxiliary constructions that equip the drainage system are used to determine the boundary conditions for the flow problems in gradually varied permanent regimes.

A mathematical model was developed in this study for the simple junction chamber (with or without slope change/breaking; with or without change of shape and/or dimensions of the cross section of the drainage pipes/culverts).

For the mathematical modeling we took into account the local charge losses when sudden change of flow section occurs, as well as the equations belonging to the theorem of the motion quantity (impulse).

In a permanent regime, we define the *control surface* as being the fixed surface **S** that limits a space **D** belonging to the space occupied by a fluid in motion.

If on the surface **S** there are rigid solid bodies, their surfaces must be considered as belonging to the control surface.

In a permanent regime, according to the *motion quantity (impulse) theorem*, the motion quantity that goes through the control surface **S** in a time unit, is equal to the sum of the exterior forces exerted on the fluid in the space **D** limited by the surface S (Ionescu, D. Gh., 1997):

$$\int_S \rho \cdot \vec{v} \cdot (\vec{v} \cdot \vec{n}) \cdot \mathrm{d}A = \sum \vec{F}_e \qquad (1)$$

where:

ρ=fluid density in the space **D**;

$\vec{v}$ = velocity vector on surface **S**;

$\vec{n}$ = exterior unit normal on surface **S**.

Usually, the control surface **S** is made of:

$$\mathbf{S} = \mathbf{S_I} + \mathbf{S_E} + \mathbf{S_L} + \mathbf{S_S} \qquad (2)$$

where:

$\mathbf{S_I}$ – the entering surface in the space **D**;

$\mathbf{S_E}$ – the exit surface in the space **D**;

$\mathbf{S_L}$ – the surface of the space **D** bordered by the free level of the water;

$\mathbf{S_S}$ – the surface of the space **D** bordered by solid bodies (walls, immersed bodies).

The sum of the exterior forces $\sum \vec{F}_e$, has the following general expression, (3) (Strecker W. and Wayne C. Hub, 2002):

$$\sum \vec{F}_e = \vec{P}_I + \vec{P}_E + \vec{P}_L + \vec{P}_S + \vec{F}_g + \vec{F}_r \qquad (3)$$

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where:

$\vec{P}_I, \vec{P}_E, \vec{P}_L, \vec{P}_S$ =pressure forces that act normally, on the respective surfaces $\mathbf{S_I}$, $\mathbf{S_E}$, $\mathbf{S_L}$ and $\mathbf{S_S}$;

$\vec{F}_g$ =the fluid gravity in the space **D**, pointing downwards;

$\vec{F}_r$ =the resistance force corresponding to the losses of hydraulic charge, h_r (usually, losses h_r are mainly local, caused by the singularities **σ** between the entrance sectors „in" and the exit ones „from" the space **D**), opposing the flow direction and usually horizontal.

The absolute value of the pressure force, *P*, which acts on the vertical surface **S**, can be assessed using the following general relation (4):

$$P = g \cdot \rho \cdot h_G \cdot A = \gamma \cdot h_G \cdot A \tag{4}$$

where:

g=gravity acceleration;

$\gamma = g \cdot \rho =$ specific gravity of the fluid (water);

A= the area of the surface **S**; h_G=the depth of the centroid of the surface **S**.

The absolute value of the gravity F_g of the fluid in the space **D** is given by the following expression (5):

$$F_g = g \cdot \rho \cdot W = \gamma \cdot W \tag{5}$$

Where:

W= is the volume of the fluid in the space **D**.

The modulus of the resistance force F_r of (6):

$$F_r = \frac{\gamma \cdot Q \cdot h_r}{V_r} = \frac{\gamma \cdot Q \cdot h_r}{Q/A_r} = \gamma \cdot A_r \cdot h_r \tag{6}$$

where:

Q=the flow rate of the stream of fluid between the sectors ***i-i*** and ***e-e***, which limit the singularity **σ**;

V_r= the average velocity of the stream between the sectors ***i-i*** and ***e-e***;

A_r= the average area of the live section of the stream between the sectors ***i-i*** and ***e-e***.

At the point of sudden change of the flowing section, depending on the ratio between the areas of the sectors ***i-i*** and ***e-e***, the local charge losses h_r can be determined using one of the following relations:

- for the sudden widening of the section, $h_r = h_r^{Lb}$,

$$h_r^{Lb} = \left(1 - \frac{A_i}{A_e}\right)^2 \cdot \frac{V_i^2}{2 \cdot g} \tag{7}$$

- for the sudden narrowing of the section, $h_r = h_r^{Ib}$,

$$h_r^{Ib} = \frac{1}{2} \cdot \left(1 - \frac{A_e}{A_i}\right) \cdot \frac{V_e^2}{2 \cdot g} \tag{8}$$

where:

A_i, A_2 = the area of the live section in the input sectors ***i-i*** and, respectively, output ***e-e***;

V_i, V_2 =the average velocity in the input sectors ***i-i*** and, respectively, output ***e-e***.

For a stream channel, limited by a rigid side surface $\mathbf{S_S}$, calculating the surface integral from the left side, the expression (1) becomes:

$$\rho \cdot Q \cdot \left(\beta_E \cdot \vec{V}_E - \beta_I \cdot \vec{V}_I\right) = \sum \vec{F}_e \tag{9}$$

where:

Q=the input/output flow rate into/out of the space **D**

β;$_I$, β_E = the motion quantity coefficient on the surface $\mathbf{S_I}$ and, respectively, $\mathbf{S_E}$;

V_I, V_E = the average velocity on the surface $\mathbf{S_I}$ and, respectively, $\mathbf{S_E}$; these can be rendered by using the continuity equation, (10):

$$V = Q/A \tag{10}$$

MATERIAL AND METHOD

The chambers of this type are usually located in the central sectors of the drain/culvert, if one or more of the following conditions are met: 1° – the change of slope of the culvert bed (basemat); 2° – break in slope; 3° change of shape and/or size of the cross-section of the pipes/culverts.

In order to apply the motion quantity theorem for a junction chamber (of parallelipiped shape) of two collector sectors, we designed the calculus formula depicted in *(figure 1)*. The control space **D** includes: I° – the downstream section of the upstream collector sector, limited upstream by the surface $\mathbf{S_r}$; II° – the actual chamber and III° – the upstream section of the downstream collector, limited downstream by the surface $\mathbf{S_E}$. The sections I° and III° are long enough so that we can consider the hydrostatic distribution of pressures on the surfaces $\mathbf{S_r}$ and $\mathbf{S_E}$, but, at the same time, short enough so that we can neglect the distributed charge losses and the grade variation of the drain bed (the basemats of the sections I° and III° are considered to be horizontal).

Both sectors, I° and III°, transport the same flow, *Q* we only selected the geometrical characteristics of area and elevation of the centroid for the live section of the stream:

- For the sector I°:

A_1=f_{A1} (h), z_{G1}=f_{zG1} (h) (11)

and for the sector II°,

A_2=f_{A2} (h), z_{G2}=f_{zG2} (h) (12)

For the calculus formula that was adopted *(fig. 1),* the expression (3) is re-written in extenso in the following way:

$$\sum \vec{F}_e = \vec{P}_I + \vec{P}_E + \sum \vec{P}_L + \sum \vec{P}_S + \sum \vec{F}_g + \vec{F}_r \quad (13)$$

Applying the relation (9), designed following the flowing direction (x-x axis), considering the directions of the exterior forces (13), we get:

$$\rho \cdot Q \cdot \left(\beta_E \cdot V_E - \beta_1 \cdot V_1\right) = \sum \left(\vec{F}_e\right)_x \quad (14)$$

$$\sum \left(\vec{F}_e\right)_x = P_I - P_E + \left(P_{SC1} - P_{SC2}\right) - F_r \quad (15)$$

where: P_{SC1} and P_{SC2} are the exterior forces (as reactions of the hydrostatic pressures) which act, respectively, on the upstream and downstream walls of the chamber.

Next, using the relations (10), (11) and (12), the velocities V_I and V_E can be written in this way:

$$V_I = \frac{Q}{f_{A1}\ (h_1)} \quad \text{and} \quad V_E = \frac{Q}{f_{A2}\ (h_2)} \quad (16)$$

The pressures P_I, P_E , P_{SC1} and P_{SC2} are determined using the expressions of the type (4), in which the area A and the centroid elevation, z_G, are evaluated using the relations (11) and (12):

$$P_I = \gamma \cdot \left(h_1 - z_{G1}\right) \cdot A_1 = \gamma \cdot \left(h_1 - f_{zG1}(h_1)\right) \cdot f_{A1}(h_1) \quad (17)$$

$$P_E = \gamma \cdot \left(h_2 - z_{G2}\right) \cdot A_2 = \gamma \cdot \left(h_2 - f_{zG2}(h_2)\right) \cdot f_{A2}(h_2) \quad (18)$$

$$P_{SC1} = \gamma \cdot \left\{ \frac{1}{2} \cdot l \cdot \left(z_C - z_{RC}\right)^2 - f_{A1}\left(z_C - z_{R1}\right) \cdot \left[z_C - f_{zG1}\left(z_C - z_{R1}\right)\right] \right\} \quad (19)$$

$$P_{SC2} = \gamma \cdot \left\{ \frac{1}{2} \cdot l \cdot \left(z_C - z_{RC}\right)^2 - f_{A2}\left(z_C - z_{R2}\right) \cdot \left[z_C - f_{zG2}\left(z_C - z_{R2}\right)\right] \right\} \quad (20)$$

From the equations (19) and (20) results (21):

$$P_{SC1} - P_{SC2} = \gamma \cdot f_{A2}\left(z_C - z_{R2}\right) \cdot \left[z_C - f_{zG2}\left(z_C - z_{R2}\right)\right] - \\ -\gamma \cdot f_{A1}\left(z_C - z_{R1}\right) \cdot \left[z_C - f_{zG1}\left(z_C - z_{R1}\right)\right] \quad (21)$$

The resistance force F_r can be determined with the expression (6), where:

$$A_r = l \cdot \left(z_C - z_{RC}\right) \quad (22)$$

$$h_r = h_r^{Lb} + h_r^{Ib} = \left(1 - \frac{A_1}{A_r}\right)^2 \cdot \frac{V_I^2}{2 \cdot g} + \frac{1}{2} \cdot \left(1 - \frac{A_2}{A_r}\right) \cdot \frac{V_E^2}{2 \cdot g} \quad (23)$$

Next, using the expressions (11) and (12) and and considering the expressions (16) for the velocities V_I and V_E from the expressions (6), (7) and (22) it results the following expression for the force F_r:

$$F_r = \rho \cdot \frac{l}{2} \cdot \left(z_C - z_{RC}\right) \cdot Q^2 \cdot \\ \cdot \left\{ \left[1 - \frac{f_{A1}\ (h_1)}{l \cdot \left(z_C - z_{RC}\right)}\right]^2 \frac{1}{\left[f_{A1}\ (h_1)\right]^2} + \frac{1}{2}\left[1 - \frac{f_{A2}\ (h_2)}{l \cdot \left(z_C - z_{RC}\right)}\right] \frac{1}{\left[f_{A2}\ (h_2)\right]^2} \right\} \quad (24)$$

Finally, considering the relations (16)÷(18), (21) and (24), the motion quantity equation (14) can be re-written in this way:

$$\Lambda\left(h_1, h_2, z_C\right) = 0 \quad (25)$$

that is an equation in the variables h_1, h_2 and z_C.

As far as the elevation of the free water level in the chamber is concerned, z_C, in reality we can come across one of the two following situations: the elevation z_C is monitored, so it has a known value, $z_C = \tilde{z}_C$; while in the absence of monitoring, the elevation z_C can be approximated in this way:

$$z_C = \frac{z_1 + z_2}{2} = \frac{z_{R1} + h_1 + z_{R2} + h_2}{2} \quad (26)$$

Hence, in both aforementioned situations, the only variables from the equation (26) are the depths h_1 and h_2; depending on the direction on the axis s when making the hydraulic calculus – in the flowing direction (from upstream to downstream) or in the opposite direction to the flow (from downstream to upstream) – in the equation (25), the depth h_1 is known and h_2 unknown, or, respectively, the depth h_2 is known and h_1 unknown.

RESULTS AND DISCUSSION

The mathematical model described by the equations (11)÷(26) was applied to a simple junction chamber characterized by the following determining dimensions *(table 1):* upstream sector CL 2800/2400, with z_{R1}=40.00 m*;* downstream sector CL 2500/2150 with z_{R2} =39.80 m; prismatic chamber with z_{RC} =38.50 m , L= 3.00 m*,* l =6.5 m, ($\beta_1 = \beta_2$=1.10; $\alpha_1 = \alpha_2$=1.30).

a). For the flow rate Q=5.30 m³/s and h_1=h_{am}=2.00 m, we got:

Z_1 = 42.00 m, A_1 =4.5844 m², Z_{G1} = 41.0774 m, V_1 =1.1561 m/s, P_1 = 41494 N.

The equation (25), with the condition (26), was numerically solved through the Nelder-Mead algorithm, with the starting value $(h_2)_0$=1.99 m and the maximum allowed error of 10^{-5} m, and, after 47 iterations, we got the solution h_2 =h_{av}=1.6435 m.

The solution $h_2 = h_{av} = 1.6435$ m was achieved for the following values of the hydraulic-functional parameters of the chamber:

Z_2 = 41.4435 m, Z_C = 41.7218 m, A_2 = 3.3881m^2, V_2 =1.5643 m/s
$P_E = P_2$ = 24598 N, Z_{G2} = 40.7034 m, Z_C = 41.7218 m,
$\mathrm{f}_{zG1}\left(z_C - z_{R1}\right) = Z_{cG1}$ = 40.9574 m, A_C = 20.9414 m^2, V_C = 0.2531 m/s,
$P_{SC1} = P_{C1}$ = 301140 N,
A_{2C} = 3.8765 m^2, $\mathrm{f}_{zG2}\left(z_C - z_{R2}\right) = Z_{cG2}$ = 40.8133 m, $P_{SC2} = P_{C2}$ = 296380 N,
h_r^{Lb} = hr_Lb = 0.0416 m, h_r^{Ib} = hr_Ib = 0.0523 m, $F_r = Ff$ = 19277 N.

b). The calculus method presented above was repeatedly applied for the following values of the functional parameters:

$$Q \in \{Q_i\} = \{Q_1, Q_2, Q_3\} = \{5.30, 7.50, 10.00\}$$

, $h_1 = h_{am} \in \left(h_{am}^{\min}\left(Q_i\right), h_{am}^{\max}\right)$

and with the condition:

$$h_2 = h_{av} \in \left(h_{av}^{\min}\left(Q_i\right), h_{av}^{\max}\right).$$

where the values $h_{am}^{\max}$ and $h_{av}^{\max}$ correspond to the depth for which the flow modulus is maximum.

The values $h_{am}^{\min}\left(Q_i\right)$ and $h_{av}^{\min}\left(Q_i\right)$ are equal to the critical depths, $h_{am}^{CR}\left(Q_i\right)$ and $h_{av}^{CR}\left(Q_i\right)$, corresponding to the flow rate Q_i, and are the solutions of the equations of the type

$$\frac{A_{CR}^3}{B_{CR}} - \frac{\alpha \cdot Q_i^2}{g} = 0$$

In this way we got the graphic representations from *(fig. 2)* ; the coordinates point $(h_{am}, h_{av})_{\mathrm{Test}}$ =(2.00 m, 1.6435 m), determined through the calculus detailed in section **a)** and emphasized by the marker „*pentagram*”, is correctly located on the curve with the parameter Q_1=5.30 m^3/s.

CONCLUSIONS

The notion of hydraulic-functional characteristic of the junction chambers of the drains and culverts was introduced, describing the relation between the depths of the water from upstream and downstream from the chamber for a certain value of the transported flow rate; when a lot of discrete values are allocated to the flow rate, the relation between the two depths is described by a family of hydraulic-functional characteristic curves.

A mathematical model was developed for a family of hydraulic-functional characteristic curves for the simple junction chamber (with or without change/break in slope; with or without changing the shape and/or sizes of the cross-section).

The mathematical model elaborated in section 2 was applied using an adequate computer software program for a simple junction chamber with break in slope and a change of shape and the sizes of the cross-section on a pipeline, and the resulting family of hydraulic-functional characteristic curves was graphically presented.

The abovementioned mathematical model and computer program may be implemented into complex software packages applicable for any of the permanent – gradual or varied flowing regime.

Table 1

The determination of the values $h_{am}^{\max}$, $h_{av}^{\max}$, $h_{am}^{\min}\left(Q_i\right)$ and $h_{av}^{\min}\left(Q_i\right)$

Section	Tube	$h_{am}^{\max}/h_{av}^{\max}$ [m]	Q_i [m^3/s]	$\alpha \cdot Q_i^2/g$ [m^5]	B_{CR} [m]	A_{CR} [m^2]	A_{CR}^3/B_{CR} [m^5]	$h_{am}^{\min}/h_{av}^{\min}$ [m]
1	2	3	4	5	6	7	8	9
upstream	CL 2800/2400	2.2448	5.30	3.7224	2.7982	2.1839	3.7224	1.0496
			7.50	7.4541	2.7554	2.7386	7.4541	1.2490
			10.00	13.2518	2.6532	3.2760	13.2518	1.4473
downstream	CL 2500/2150	2.0098	5.30	3.7224	2.4737	2.0960	3.7224	1.0808
			7.50	7.4541	2.3748	2.6062	7.4541	1.2907
			10.00	13.2518	2.1977	3.0767	13.2518	1.4958

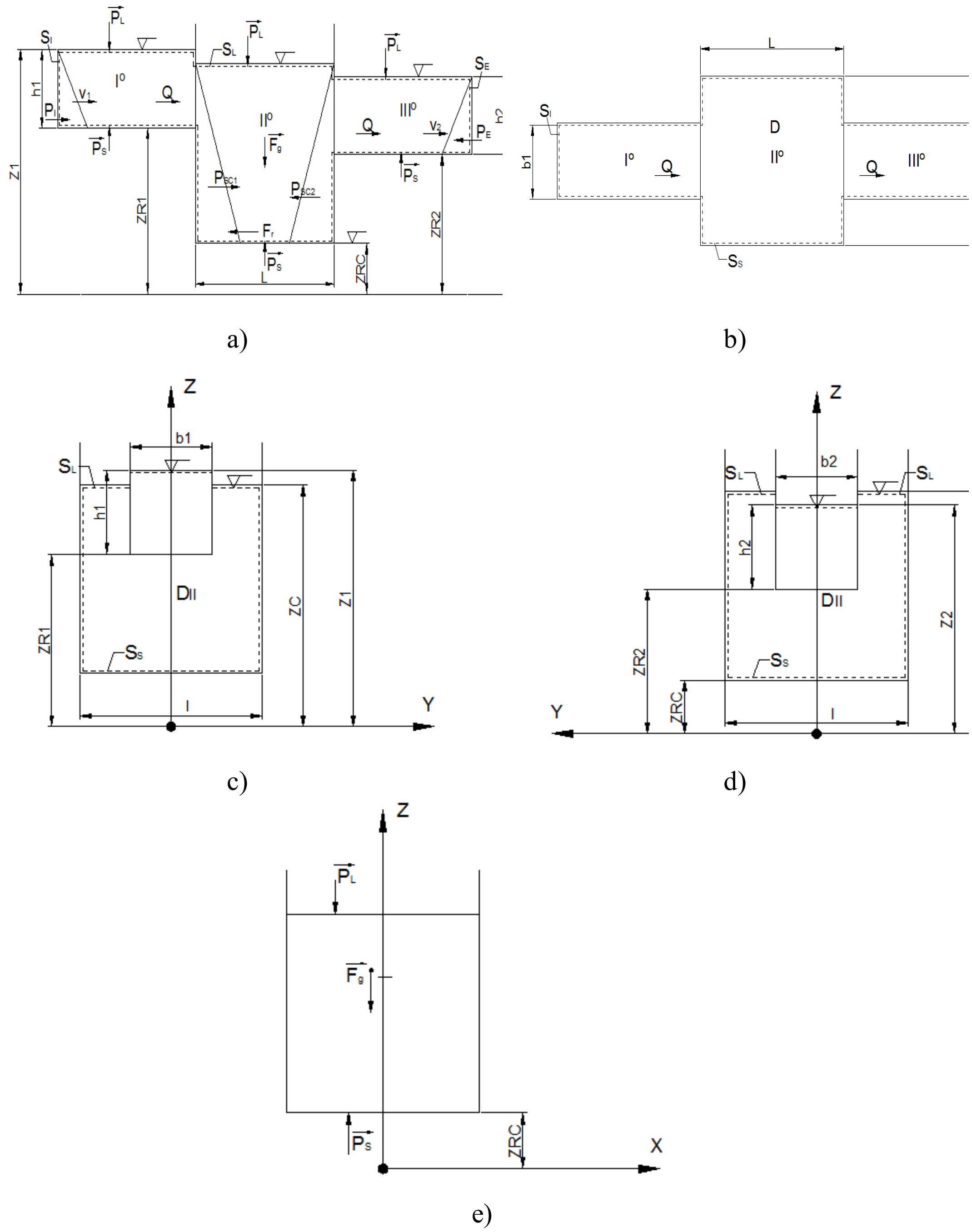

Figure 1 **The calculus formula for a simple junction chamber:**

a). Longitudinal profile; b) Plane view (horizontal); c) Upstream wall of the chamber; d) downstream wall of the chamber; e) back-lateral wall of the chamber (the lateral-front wall of the chamber differs only in the contrary direction for the *x* axis). b_1, b_2 = the widths of the sectors I° and III°; *L*, *l* =length and width of the chamber; z_{R1}, z_{R2} = elevations of the section beds I° and III°; z_{RC} = elevation of the chamber bottom; z_1, z_2 = elevations of the free water levels for the sectors I° and III°; z_C = elevation of the free water level in the chamber

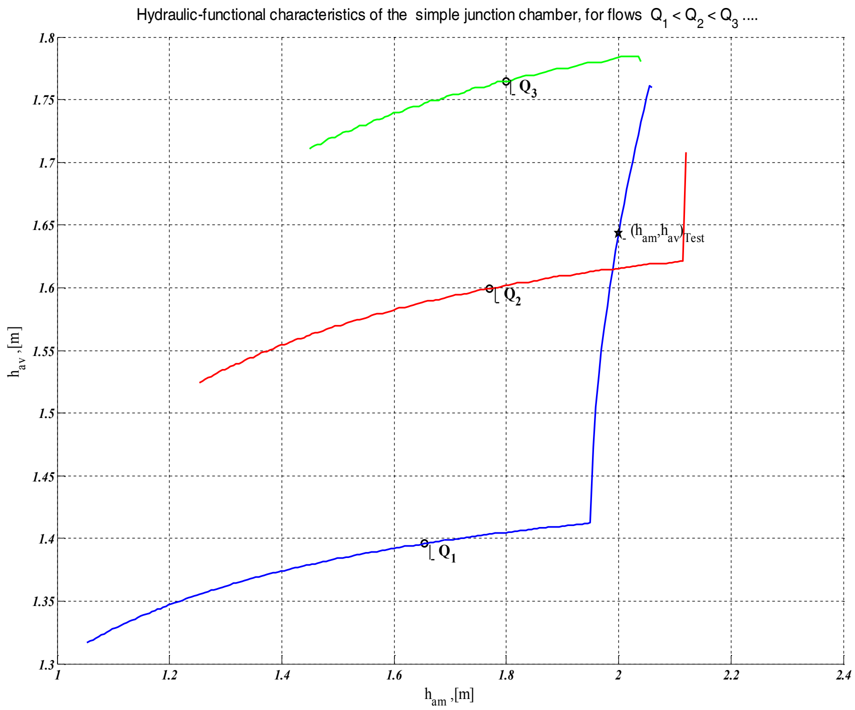

Figure 2. **The hydraulic-functional characteristics of the simple junction chamber with the following determining dimensions: upstream sector CL..; downstream sector CL , prismatic chamber with z_{RC} =38.50 m , *L*= 3.00 m, *l* =6.5 m.**

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