COLORIMETRIC CHARACTERISATION OF TRICHROMATIC RGB DIGITAL IMAGES. II. CIE COLORIMETRIC CHARACTERISTICS

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Abstract

In performing the colour analysis of a particular colour, according to the CIE standards, the CIE xy, CIE ab and CIE uv colour diagrams are used, as derived from the corresponding XYZ, Lab and Luv colour spaces. They enable the quantification of features such as hue, lightness, saturation, and chromaticity, which are useful in tinctorial practice. As regards the basic features, based on three models included in a structural unit, the colorimetric characteristics are analysed and evolving trends are presented. For systems with a reference state (standard illuminant D65), colour difference computing alternatives are provided. An analysis of colour assessment criteria is provided and, drawing on its conclusions, the appropriate representation space is recommended.

Keywords: CIE XYZ, CIE Yxy, CIE xy, CIE LAB, CIE $L^* a^* b^*$, CIE $L^* u^* v^*$, CIE ΔE^*_{ab}-1976, CIE ΔE^*_{uv}-1994, CIE ΔE^*-2000.

In performing the colour analysis of a particular colour, according to the CIE standards, the CIE xy, CIE ab and CIE uv colour diagrams are used, as derived from the corresponding XYZ, Lab and Luv colour spaces. They enable the quantification of features such as hue, lightness, saturation, and chromaticity, which are useful in tinctorial practice. As regards the basic features, based on three models included in a structural unit, the colorimetric characteristics are analysed and evolving trends are presented. For systems with a reference state (standard illuminant D65), colour difference computing alternatives are provided. An analysis of colour assessment criteria is provided and, drawing on its conclusions, the appropriate representation space is recommended.

MATERIAL AND METHOD

CIE XYZ (*CIE – International Commission on Illumination / Commission Internationale d'Eclairage*)

Each point corresponding to a colour in the CIE XYZ colour space or the imaginary locus corresponding to the CIE xy is determined using the general algorithm which includes the known values: reflection / transmission spectral curve, energy feature of the illuminant and the standard composition functions of colours.

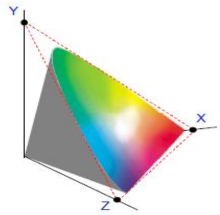

Figure 1: **The position of a colour C in the CIE XYZ imaginary space**

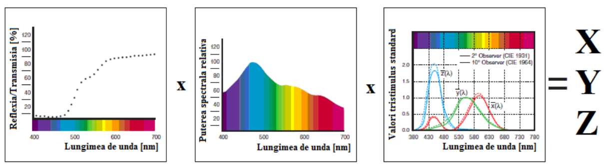

Figure 2: **Computation elements for values X,Y,Z**

$$X = k \cdot \int_{\lambda=380\,nm}^{780} \Phi(\lambda) \cdot \bar{x}(\lambda) \cdot \tau(\lambda) \cdot d\lambda,$$

$$Y = k \cdot \int_{\lambda=380\,nm}^{780} \Phi(\lambda) \cdot \bar{y}(\lambda) \cdot \tau(\lambda) \cdot d\lambda,$$

$$Z = k \cdot \int_{\lambda=380\,nm}^{780} \Phi(\lambda) \cdot \bar{z}(\lambda) \cdot \tau(\lambda) \cdot d\lambda;$$

where:

$$k = \frac{100}{\int_{380}^{780} \Phi(\lambda) \cdot \bar{y}(\lambda) \cdot d\lambda}$$, normalisation factor (normalisation coefficient),

$\Phi(\lambda)$ – the relative spectral distribution of the standard light source energy (illuminants A, B, C, D, E (evenly distributed), F_1 – F_{12} – fluorescents);

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$\bar{x}(\lambda), \bar{y}(\lambda), \bar{x}(\lambda)$ – standard colour matching functions;

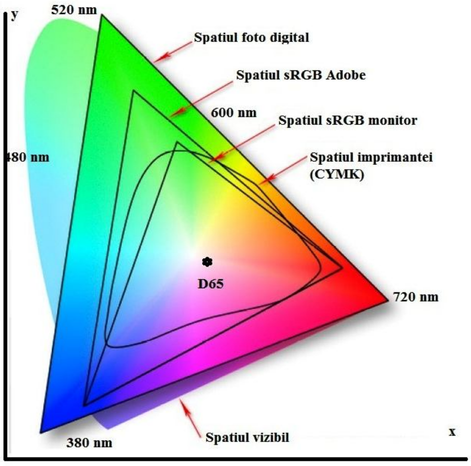
$\tau(\lambda)$ – spectral reflexion/transmission coefficient;

CIE xy

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{X+Y+Z} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix};$$

$$\begin{bmatrix} x \\ y \\ Y \end{bmatrix} = \begin{bmatrix} \dfrac{X}{X+Y+Z} \\ \dfrac{Y}{X+Y+Z} \\ Y \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{Y}{y} \cdot \begin{bmatrix} x \\ y \\ 1-x-y \end{bmatrix}.$$



Digital photography space
Adobe sRGB colour space
Monitor sRGB colour space
Printer's colour space (CYMK)
Visible colour space

Figure 3: **Locus CIE xy and various gamuts**

RESULTS AND DISSCUTIONS

CIE L*a*b* 1976

The *L*a*b** colour system is used as a standard in: colour management, colour reproduction through photography, colour copying, colour printing, etc.

The L*a*b * model is based on the theory that a colour cannot be both green and red or yellow and blue. *Cartesian coordinates are used in the L*a*b* space.*

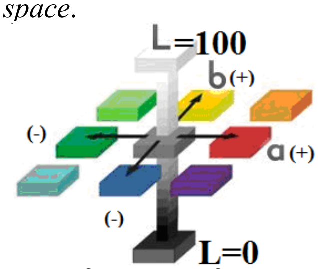


Figure 4: **Meaning of colour shifts in the L*a*b* space**

a(-) – greener, a(+) – redder,
b(+) – more yellow, b(-) = bluer,
white at L=100 and black at L=0.

The **L*a*b*** space is much more uniform than XYZ space and is recommended for coloured surfaces.

$$L^* = \begin{cases} open: \\ 116 \cdot \left(\dfrac{Y}{Y_0}\right)^{\frac{1}{3}} - 16, & \left(\dfrac{Y}{Y_0}\right) > 0{,}008856 \\ closed: \\ 903{,}3 \cdot \left(\dfrac{Y}{Y_0}\right), & \left(\dfrac{Y}{Y_0}\right) \le 0{,}008856 \end{cases}$$

$$a^* = 500 \cdot \left(f\left(\frac{X}{X_0}\right) - f\left(\frac{Y}{Y_0}\right) \right)$$

$$b^* = 200 \cdot \left(f\left(\frac{Y}{Y_0}\right) - f\left(\frac{Z}{Z_0}\right) \right)$$

$$f(x) = \begin{cases} x^{\frac{1}{3}}, & x > 0{,}008856 \\ 7{,}787 \cdot x + \dfrac{16}{116}, & x \le 0{,}00856 \end{cases}$$

$$f(x) = \left(\frac{X}{X_0}\right)^{\frac{1}{3}}; \left(\frac{Y}{Y_0}\right)^{\frac{1}{3}} \left(\frac{Z}{Z_0}\right)^{\frac{1}{3}}$$

X_0, Y_0, Z_0 refer to the CIE XYZ values of the standard illuminant, for instance D65.

L* values range between 0 and 100.

Non-linear relations for L*, a* and b* are intended to imitate the eye's logarithmic response.

The reverse transformation:

$$Y_1 = \frac{L+16}{116}$$

$$X_1 = \frac{a^*}{500} + Y_1$$

$$Z_1 = -\frac{b^*}{200} + Y_1$$

$$X_1 = \begin{cases} X_1^3, & X_1 > 0.206893 \\ \dfrac{\left(X_1 - \dfrac{16}{116}\right)}{7.787}, & X_1 \le 0.206893 \end{cases}$$

$$Y_1 = \begin{cases} Y_1^3, & Y_1 > 0.206893 \\ \dfrac{\left(Y_1 - \dfrac{16}{116}\right)}{7.787}, & Y_1 \le 0.206893 \end{cases}$$

$$Z_1 = \begin{cases} Z_1^3, & Z_1 > 0.206893 \\ \dfrac{\left(Z_1 - \dfrac{16}{116}\right)}{7.787}, & Z_1 \le 0.206893 \end{cases}$$

$$X = X_0 \cdot X_1$$
$$Y = Y_0 \cdot Y_1$$
$$Z = Z_0 \cdot Z_1.$$

HUNTER Lab

$$L = 100 \cdot \sqrt{\frac{Y}{Y_0}}$$

$$a = K_a \cdot \left(\frac{\frac{X}{X_0} - \frac{Y}{Y_0}}{\sqrt{\frac{Y}{Y_0}}} \right)$$

$$b = K_b \cdot \left(\frac{\frac{Y}{Y_0} - \frac{Z}{Z_0}}{\sqrt{\frac{Y}{Y_0}}} \right).$$

K_a, K_b – the chromaticity coefficients for the selected standard illuminant.
X, Y, Z – CIE tristimulus values,
X_0, Y_0, Z_0 – the illuminant specific tristimulus values,
$Y_0 = 100$.

The values of the CIE D_{65} illuminant:

	X_0	Z_0	K_a	K_b
CIE 1931 2^0	95.02	108.82	172.30	67.20
CIE 1964 10^0	94.83	107.38	172.10	66.70

CIE LAB

In the CIE Lab space the following transformation matrix is used:

$$\begin{bmatrix} L \\ A \\ B \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Reverse transformation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} L \\ A \\ B \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = L \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + A \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + B \cdot \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = L \cdot c + A \cdot a + B \cdot b$$

The points of vector a are in direction X, the points of vector b in the negative direction on the Z axis, while vector c points are along the diagonal in the XYZ space

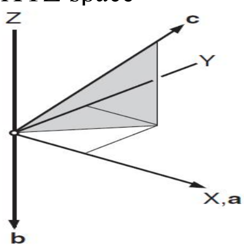


Figure 5: XZY – **LAB spatial transformation**

CIE Yuv (1960)

The CIE Yxy space is transformed into the CIE Yuv in order to represent colours having the same magnitude, therefore in an even space, which is not sufficient however.

$$\begin{cases} u = \dfrac{2 \cdot x}{(-x + 6 \cdot y + 1.5)} \\ v = \dfrac{3 \cdot y}{(-x + 6 \cdot y + 1.5)} \end{cases}.$$

or

$$\begin{cases} u = \dfrac{4 \cdot X}{X + 15 \cdot Y + 3 \cdot Z} = \dfrac{4 \cdot x}{-2 \cdot x + 12 \cdot y + 3} \\ v = \dfrac{6 \cdot Y}{X + 15 \cdot Y + 3 \cdot Z} = \dfrac{6 \cdot y}{-2 \cdot x + 12 \cdot y + 3} \end{cases}.$$

CIE Yu'v'(1976)

$$\begin{cases} u' = u = \dfrac{2 \cdot x}{(-x + 6 \cdot y + 1.5)} \\ v' = 1.5 \cdot v = \dfrac{4.5 \cdot y}{(-x + 6 \cdot y + 1.5)} \end{cases}.$$

CIE L^* u'v' 1976

In another variant, lightness can be represented in relation to the coordinates:

$$\begin{cases} u' = \dfrac{4 \cdot X}{X + 15 \cdot Y + 3 \cdot Z} = \dfrac{4 \cdot x}{-2 \cdot x + 12 \cdot y + 3}, \\ v' = \dfrac{9 \cdot Y}{X + 15 \cdot Y + 3 \cdot Z} = \dfrac{9 \cdot y}{-2 \cdot x + 12 \cdot y + 3}, \end{cases}$$

for the **Lu′v′** space and the reverse transformation:

$$\begin{cases} x = \dfrac{27 \cdot u'}{18 \cdot u' - 48 \cdot v' + 36} = \dfrac{9 \cdot u'}{6 \cdot u' - 16 \cdot v' + 12}; \\ y = \dfrac{12 \cdot v'}{18 \cdot u' - 48 \cdot v' + 36} = \dfrac{4 \cdot v'}{6 \cdot u' - 16 \cdot v' + 12} \end{cases},$$

where x and y are the values in the CIE xy – 2^0 space, illuminant C with values: u_0'=0,2009 and v_0'=0,4610.

CIE L*u*v*

The CIE $L^*u^*v^*$ is appropriate for characterising the coloured light of monitors, TVs etc.

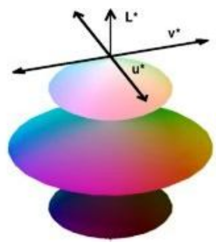


Figure 6: **CIE L*u*v***

For the **CIE $L^*u^*v^*$** space**:**

$$L^* = \begin{cases} \left(\dfrac{29}{3}\right) \cdot \dfrac{Y}{Y_0}, for \left(\dfrac{Y}{Y_0}\right) \le \left(\dfrac{6}{29}\right)^3; \\ 116 \cdot \left(\dfrac{Y}{Y_0}\right)^3 - 16, for \left(\dfrac{Y}{Y_0}\right) > \left(\dfrac{6}{29}\right)^3, \end{cases}$$

$$\begin{cases} u^* = 13 \cdot L^* \cdot (u' - u'_0), \\ v^* = 13 \cdot L^* \cdot (v' - v'_0). \end{cases}$$

The reverse relations:

$$\begin{cases} u' = \dfrac{u^*}{13 \cdot L^*} + u'_0, \\ v' = \dfrac{v^*}{13 \cdot L^*} + v'_0. \end{cases}$$

$u^{'}, v^{'}$ – the values in the CIE $u^{'}v^{'}$ colour space.

$$Y = \begin{cases} Y_0 \cdot L^* \cdot \left(\frac{3}{29}\right)^3, for(L^* \le 8), \\ Y_0 \cdot \left(\frac{L^* + 16}{116}\right)^3, for(L^* > 8), \end{cases}$$

$$X = Y \cdot \frac{9 \cdot u^{'}}{4 \cdot v^{'}},$$

$$Z = Y \cdot \frac{12 - 3 \cdot u^{'} - 20 \cdot v^{'}}{4 \cdot v^{'}}, for(L^* < 8)$$

and

$$X = -9 \cdot Y \cdot \frac{u'}{((u' - 4) \cdot v' - u'v')}$$

$$Z = (9 \cdot Y - 15 \cdot v' \cdot Y - v' \cdot X)/(3 \cdot v'), for(L^* > 8).$$

X, Y, Z in the CIE XYZ space.

The cylindrical representation version includes the elements:

$$C^*_{ab} = \sqrt{(a^*)^2 + (b^*)^2},$$

$$h^*_{ab} = \tan^{-1}\left(\frac{b^*}{a^*}\right).$$

The $L^*a^*b^*$ diagram may be used concurrently with the $L^*u^*v^*$ diagram.

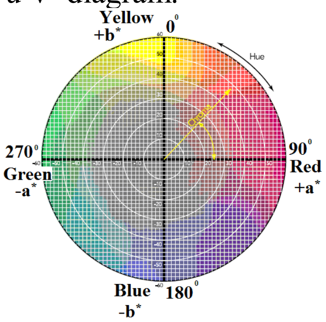


Figure 7: **The graph of the CIE $L^*a^*b^*$ to CIE L^*ch_{uv} transfer with L* = constant.**

CIE L^*ch_{uv}

L^*ch_{uv} represents the cylindrical version of the CIE $L^*u^*v^*$ space where C^*_{uv} stands for the chroma, while h^*_{uv} stands for saturation.

$$C^*_{uv} = \sqrt{(u^*)^2 + (v^*)^2},$$

$$h^*_{uv} = \tan^{-1}\left(\frac{v^*}{u^*}\right).$$

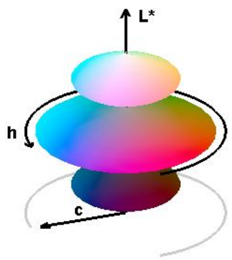


Figure 8: **CIE L^*ch space**

On the other hand, the correlation of saturation may be defined through the formula:

$$s_{uv} = \frac{C^*}{L^*} = 13 \cdot \sqrt{(u^{'} - v^{'}_0)^2 + (v^{'} - v^{'}_0)^2}.$$

CIE L*a*b*, L*u*v*and L*c*h* are non-linear functions in relation to XYZ. The L* components is linked to lightness perception. The other two components describe the chroma (C^*_{ab} or C^*_{uv}).

To calculate the differences the formula below is used:

$$\Delta H^*_{ab} = \sqrt{(\Delta E^*_{ab})^2 - (\Delta L^*_{ab})^2 - (\Delta C^*_{ab})^2};$$

$$\Delta H^*_{ab} = 2 \cdot \sqrt{C^*_{ab,1} \cdot C^*_{ab,2}} \cdot \sin\left(\frac{\Delta h_{ab}}{2}\right), h_{ab} < or > 0.$$

Colour Difference

CIE ΔE_{ab} 1976

The L*, a*, b* values are employed in calculating colour difference:

$$\Delta E^*_{ab,76} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2},$$

$$\Delta L^* = L^*_1 - L^*_2$$

$$\Delta a^* = a^*_1 - a^*_2$$

$$\Delta b^* = b^*_1 - b^*_2.$$

The value is depended upon the illuminant employed and the standard (CIE 1931 - 2^0 variant or the CIE 1964 - 10^0 variant).

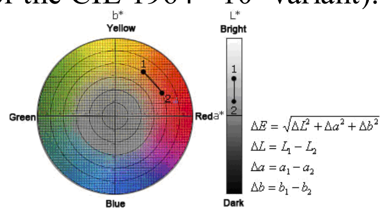


Figure 9: **CIE ΔE_{ab} 1976 calculations**

CIE ΔE_{uv} 1976

$$\Delta E^*_{uv,76} = \sqrt{(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2}$$

$$\Delta L^* = L^*_1 - L^*_2$$

$$\Delta u^* = u^*_1 - u^*_2$$

$$\Delta v^* = v^*_1 - v^*_2.$$

CIE ΔE_{ch} 1976

$$\Delta E^*_{hc} = \sqrt{(\Delta L^*)^2 + (\Delta C^*)^2 + (\Delta H^*)^2},$$

$$\Delta L^* = L^*_1 - L^*_2$$

$$\Delta C^* = C^*_1 - C^*_2$$

$$\Delta H^* = H^*_1 - H^*_2.$$

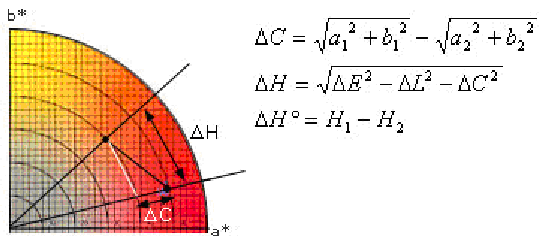


Figure 10: **CIE ΔE_{ch} 1976 calculations**

If ΔL^*, ΔC^*, ΔH^* are not equal following regular observation, the following adjustments are performed:

$$\Delta E^*_{hc} = \sqrt{\left(\frac{\Delta L^*}{l}\right)^2 + \left(\frac{\Delta C^*}{c}\right)^2 + \left(\frac{\Delta H^*}{h}\right)^2}.$$

CIE ΔE_{ab} 1994

For minor colour differences the formula below applies:

$$\Delta E^*_{ab,94} = \sqrt{\left(\frac{\Delta L_{ab}{}^*}{K_L \cdot S_L}\right)^2 + \left(\frac{\Delta C_{ab}{}^*}{K_C \cdot S_C}\right)^2 + \left(\frac{\Delta H_{ab}{}^*}{K_H \cdot S_H}\right)^2},$$

$$S_L = 1$$

$$S_C = 1 + 0.045 \cdot C^*_{ab}$$

$$S_H = 1 + 0{,}015 \cdot C^*_{ab}.$$

The weight functions S_L, S_C, S_H are dependent upon the position in the CIE $L^* a^* b^*$ space.

The parametric factors K_L, K_C, K_H with an approximate value of 1 are dependent upon on the criteria of perceptibility (texture, background, etc..) or acceptability and change with light, chroma or saturation in various industrial applications.

CIE ΔE_{ab} 2000

The formula is derived from the CIE $\Delta E_{ab,94}$ equation.

$$\Delta E^{1,2}_{ab,00} = \Delta E^{1,2}{}_{ab,00}\left(L^*_1, a^*_1, b^*_1; L^*_2, a^*_2, b^*_2\right)$$

and observes the symmetry condition:

$$\Delta E^{1,2}{}_{ab,00}\left(L^*_1, a^*_1, b^*_1; L^*_2, a^*_2, b^*_2\right) = \Delta E^{2,1}_{ab,00}\left(L^*_2, a^*_2, b^*_2; L^*_1, a^*_1, b^*_1\right)$$

$$\Delta E^*_{ab,00} = \sqrt{\left(\frac{\Delta L_{ab}{}^*}{K_L \cdot S_L}\right)^2 + \left(\frac{\Delta C_{ab}{}^*}{K_C \cdot S_C}\right)^2 + \left(\frac{\Delta H_{ab}{}^*}{K_H \cdot S_H}\right)^2 + \Delta R},$$

$$\Delta R = R_T \cdot f\left(\Delta C', \Delta H'\right) = R_T \cdot \left(\frac{\Delta C'}{k_k \cdot S_C}\right) \cdot \left(\frac{\Delta H'}{k_H \cdot S_H}\right) . \Delta$$

R – the interactive term between the chromaticity and the saturation differences. Values k_L, k_C and k_H are known. Normally: $R_T = 0$ and $S_L = 1$.

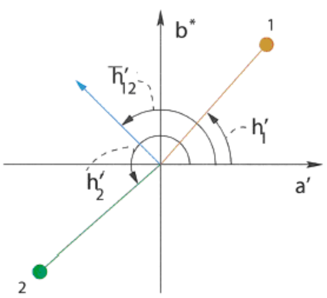


Figure 11: **Calculation of the average hue**

Stages of calculation

1. C_i', h_i' are calculated:

$$C^*_{i,ab} = \sqrt{\left(a^*_i\right)^2 + \left(b^*_i\right)^2}, i = 1{,}2$$

$$\bar{C}*_{ab} = \frac{C^*_{1,ab} + C^*_{2,ab}}{2}$$

$$G = 0.5 \cdot \left(1 - \sqrt{\frac{\left(\bar{C}*_{ab}\right)^7}{\left(\bar{C}*_{ab}\right)^7 + 25^7}}\right)$$

$$a_i' = \left(1 + G\right) \cdot a^*_i, i = 1{,}2$$

$$C_i' = \sqrt{\left(a_i'\right)^2 + \left(b^*_i\right)^2}, i = 1{,}2$$

$$h_i' = \begin{cases} 0, & b^*_i = a_i' = 0, \quad i = 1{,}2 \\ \tan^{-1}\left(b^*_i, a_i'\right) & 0 < b^*_i, a_i' > 0 \end{cases}$$

- h_i is converted from radians to degrees by multiplying the latter by $180/\pi$.

2. The following calculations are performed:

$$\Delta L' = L^*_2 - L^*_1$$

$$\Delta C' = C_2' - C_1'$$

$$\Delta h' = \begin{cases} 0, & C_1', C_2' = 0, \\ h_2' - h_1', & C_1', C_2' \neq 0, \quad \left|h_2' - h_1'\right| \leq 180^0 \\ \left(h_2' - h_1'\right) - 360, & C_1', C_2' \neq 0, \quad \left|h_2' - h_1'\right| > 180^0 \\ \left(h_2' - h_1'\right) + 360, & C_1', C_2' \neq 0, \quad \left|h_2' - h_1'\right| < -180^0 \end{cases} \quad \Delta C'$$

$$\Delta H' = 2 \cdot \sqrt{C_1' \cdot C_2'} \cdot \sin\left(\frac{\Delta h'}{2}\right).$$

and $\Delta H'$ are not absolute values and influence ΔR

3. **CIE ΔE_{ab} 2000** is calculated:

$$\bar{L}' = \frac{\left(L^*_1 + L^*_2\right)}{2}$$

$$\bar{C}' = \frac{\left(C_1' + C_2'\right)}{2}$$

$$\bar{h}' = \begin{cases} \left(h_1' + h_2'\right), & C_1', C_2' = 0, \\ \frac{\left(h_1' + h_2'\right)}{2}, & C_1', C_2' \neq 0, \quad \left|h_1' - h_2'\right| \leq 180^0 \\ \frac{\left(h_1' + h_2'\right) - 360}{2}, & C_1', C_2' \neq 0, \quad \left|h_1' - h_2'\right| > 180^0; \left(h_1' + h_2'\right) \geq 360 \\ \frac{\left(h_1' + h_2'\right) + 360}{2}, & C_1', C_2' \neq 0, \quad \left|h_1' - h_2'\right| > 180^0; \left(h_1' + h_2'\right) < 360 \end{cases}$$

$$T = 1 - 0.17 \cdot \cos\left(\bar{h}' - 30\right) + 0.24 \cdot \cos\left(2 \cdot \bar{h}'\right) +$$

$$+ 0.32 \cdot \cos\left(3 \cdot \bar{h}' + 6^0\right) - 0.2 \cdot \cos\left(4 \cdot \bar{h}' - 63\right)$$

$$\Delta\theta = 30 \cdot \exp\left\{-\left[\frac{\bar{h}' - 275^0}{25}\right]^2\right\}$$

$$R_C = 2 \cdot \sqrt{\frac{\left(\bar{C}'\right)^7}{\left(\bar{C}'\right)^7 + 25^7}}$$

$$S_L = 1 + \frac{0.015 \cdot \left(\bar{L}' - 50\right)^2}{\sqrt{20 + \left(\bar{L}' - 50\right)^2}}$$

$$S_C = 1 + 0.045 \cdot \bar{C}'$$

$$S_H = 1 + 0.015 \cdot \bar{C}' \cdot T$$

$$R_T = -\sin\left(2 \cdot \Delta\theta\right) \cdot R_C$$

$$\Delta E^*_{ab,00} = \sqrt{\left(\frac{\Delta L_{ab}^*}{K_L \cdot S_L}\right)^2 + \left(\frac{\Delta C_{ab}^*}{K_C \cdot S_C}\right)^2 + \left(\frac{\Delta H_{ab}^*}{K_H \cdot S_H}\right)^2 + \Delta R},$$

$$\Delta R = R_T \cdot f\left(\Delta C', \Delta H'\right) = R_T \cdot \left(\frac{\Delta C'}{k_k \cdot S_C}\right) \cdot \left(\frac{\Delta H'}{k_H \cdot S_H}\right).$$ Po

sition of the standard illuminant d65 in various colour spaces

CIE 1931 2°		**CIE 1964 10°**	
x_2	y_2	x_{10}	y_{10}
0.31271	0.32902	0.31382	0.33100
u'	v'	u'	v'
0,1987	0,4683	0,1979	0,4696

L	a	b
23,482	*37,384	-60,1046

Hunter ΔE

$$\Delta L = L_{sample} - L_{standard}$$

$$\Delta a = a_{sample} - a_{standard}$$

$$\Delta b = b_{sample} - b_{standard}$$

$$\Delta E = \sqrt{\Delta L^2 + \Delta a^2 + \Delta b^2}.$$

Values ΔL, Δa, Δb are positive provided the sample is more brighter, redder, and more yellow respectively than the standard sample; same values are negative if the sample is darker, greener, or bluer, respectively, than the standard sample.

Standard colorimetric analysis – CIE 1931, 1976, 1994, 2000

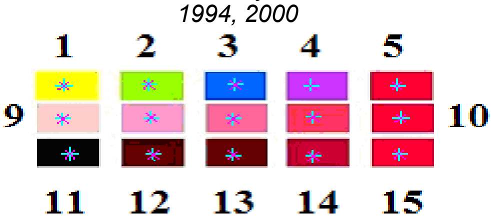


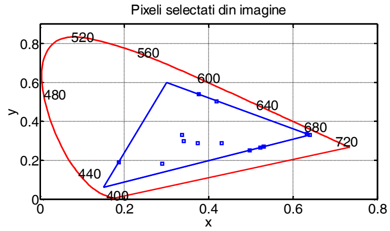
Figure 12: **WA.jpg**

RGB =
255 253 2
136 253 0
1 81 254
188 40 255
254 0 43
254 186 185
255 134 185
255 81 136
254 42 82
254 0 39
0 0 0
82 1 0
108 0 0
186 0 41
252 0 39

Msrgbxyz =
0.4125 0.3576 0.1804
0.2127 0.7152 0.0722
0.0193 0.1192 0.9502

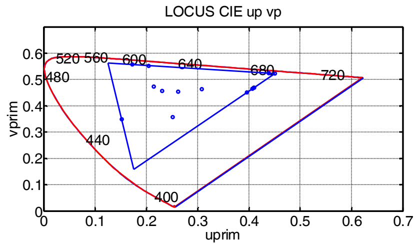
XYZ =
196.0041 235.3109 36.9865
146.5613 209.8587 32.7853
…………………………………………
84.1136 42.5157 42.5554
110.9747 56.4077 41.9310

xy =
0.4185 0.5025
0.3766 0.5392
…………………………..
0.4308 0.2880
0.5309 0.2699
NaN NaN
0.6325 0.3360
0.6400 0.3300
0.4972 0.2513
0.5302 0.2695

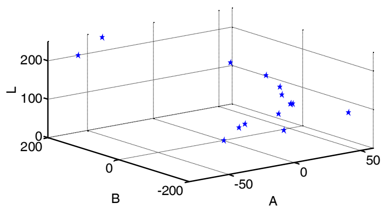


Pixels selected from image

upvp =
0.2044 0.5520
0.1728 0.5567
0.1523 0.3484
0.2512 0.3567
0.4067 0.4645
0.2143 0.4722
0.2311 0.4556
0.2623 0.4537
0.3080 0.4633
0.4102 0.4692
NaN NaN
0.4387 0.5243
0.4507 0.5229
0.3961 0.4504
0.4099 0.4688



M =
0 1 0
1 -1 0
0 1 -1

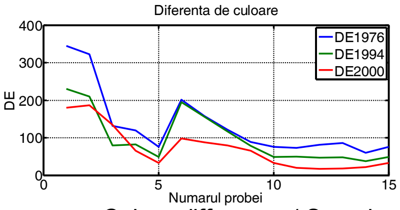


Lab =
235.3109 -39.3068 198.3245
209.8587 -63.2974 177.0735
………………………………………………
42.5157 41.5979 -0.0397
56.4077 54.5670 14.4767

Colour difference in relation to D65

WA image

DE=([DE1976 DE1994 DE2000])
345.0946 230.2285 179.7420
322.5115 210.0017 186.2778
……………………………………………..
60.2860 37.6938 22.2486
76.0501 48.7027 32.7284



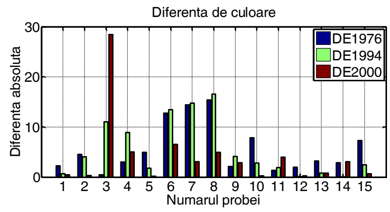
Colour difference / Sample number

Figure 13: **Colour difference**

WD image (corrected WA image)

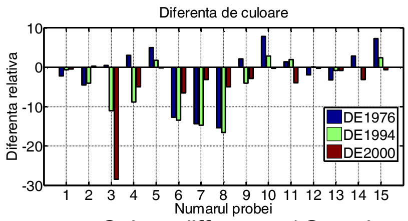
DE=([DE1976 DE1994 DE2000])

342.8389 229.5993 179.3253
318.0038 205.9812 186.5974
…………………………………………..
63.1493 37.7158 19.1515
83.3170 51.1304 32.0583



Colour difference / Sample number

Figure 14: **WA – WD absolute difference**



Colour difference / Sample number

Figure 15: **WA – WD relative difference**

CONCLUSIONS

Based on a complex research of state-of-the-art specialist literature (references 1-13), the paper provides the condensed CIE formulas for assessing colour, the underlying mathematical reasoning and graphic representation methods commonly used in practice.

The representation of colour in modern spaces - CIE $L^*a^*b^*$, CIE $L^*u^*v^*$ - aims to deliver uniform distribution.

Differences in colour rendered by the ever more complex CIE 1976, 1994, 2000 variants encompass a, b and c, h chromaticity as axes and L as the main axis.

The mathematical calculation algorithm is integrated using particular functions in the Matlab environment, which extended to processing trichromatic digital images.

Matlab extensions enable the rapid calculation of specific colour measurements.

In relation to the illuminant D65, differences emerge in the calculation of colour difference, as reflected by the absolute difference and relative difference.

Through correction (WD image), colour difference may be amplified or diminished.

The CIE 2000 calculation alternative, for image variations WA and WD, delivers values in a more limited range.

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