COLORIMETRIC CHARACTERISATION OF TRICROMATIC RGB DIGITAL IMAGES. II. CIE COLORIMETRIC CHARACTERISTICS

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Abstract

In performing the colour analysis of a particular colour, according to the CIE standards, the CIE xy, CIE ab and CIE uv colour diagrams are used, as derived from the corresponding XYZ, Lab and Luv colour spaces. They enable the quantification of features such as hue, lightness, saturation, and chromaticity, which are useful in tinctorial practice. As regards the basic features, based on three models included in a structural unit, the colorimetric characteristics are analysed and evolving trends are presented. For systems with a reference state (standard illuminant D65), colour difference computing alternatives are provided. An analysis of colour assessment criteria is provided and, drawing on its conclusions, the appropriate representation space is recommended.

Keywords: CIE XYZ, CIE Yxy, CIE xy, CIE LAB, CIE L∗ a∗ b∗, CIE L∗ u∗ v∗, CIE ΔE∗ab-1976, CIE ΔE∗uv-1994, CIE ΔE*2000.

In performing the colour analysis of a particular colour, according to the CIE standards, the CIE xy, CIE ab and CIE uv colour diagrams are used, as derived from the corresponding XYZ, Lab and Luv colour spaces. They enable the quantification of features such as hue, lightness, saturation, and chromaticity, which are useful in tinctorial practice. As regards the basic features, based on three models included in a structural unit, the colorimetric characteristics are analysed and evolving trends are presented. For systems with a reference state (standard illuminant D65), colour difference computing alternatives are provided. An analysis of colour assessment criteria is provided and, drawing on its conclusions, the appropriate representation space is recommended.

MATeRIAL AND METHOD

CIE XYZ (CIE – International Commission on Illumination / Commission Internationale d’Eclairage)

Each point corresponding to a colour in the CIE XYZ colour space or the imaginary locus corresponding to the CIE xy is determined using the general algorithm which includes the known values: reflection / transmission spectral curve, energy feature of the illuminant and the standard composition functions of colours.

\[ X = k \cdot \frac{1}{380} \int_{\lambda = 380}^{780} \Phi(\lambda) \cdot x(\lambda) \cdot \tau(\lambda) \cdot d\lambda, \]

\[ Y = k \cdot \frac{1}{380} \int_{\lambda = 380}^{780} \Phi(\lambda) \cdot y(\lambda) \cdot \tau(\lambda) \cdot d\lambda, \]

\[ Z = k \cdot \frac{1}{380} \int_{\lambda = 380}^{780} \Phi(\lambda) \cdot z(\lambda) \cdot \tau(\lambda) \cdot d\lambda; \]

where:

\[ k = \frac{100}{380} \] normalisation factor

(normalisation coefficient),

\( \Phi(\lambda) \) – the relative spectral distribution of the standard light source energy (illuminants A, B, C, D, E (evenly distributed), F1 – F12 – fluorescents);

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The \( L^*a^*b^* \) space is much more uniform than XYZ space and is recommended for coloured surfaces.

The \( L^*a^*b^* \) colour system is used as a standard in: colour management, colour reproduction through photography, colour copying, colour printing, etc.

The \( L^*a^*b^* \) model is based on the theory that a colour cannot be both green and red or yellow and blue. Cartesian coordinates are used in the \( L^*a^*b^* \) space.

\[
\begin{align*}
  x(\lambda), y(\lambda), z(\lambda) & \quad \text{standard colour matching functions; } \\
  \tau(\lambda) & \quad \text{spectral reflexion/transmission coefficient; } \\
  [x] = \frac{1}{x+y+z} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\
  [y] = \frac{X}{x+y+z} \\
  [z] = \frac{z}{x+y+z} \\
  X = \frac{y}{y+y+z} \\
  Y = \frac{x}{x+y+z} \\
  Z = \frac{1-x-y}{x+y+z}
\end{align*}
\]

Results and Discussion

CIE \( L^*a^*b^* \) 1976

The \( L^*a^*b^* \) colour system is used as a standard in: colour management, colour reproduction through photography, colour copying, colour printing, etc.

The \( L^*a^*b^* \) model is based on the theory that a colour cannot be both green and red or yellow and blue. Cartesian coordinates are used in the \( L^*a^*b^* \) space.
The CIE Yxy space is transformed into the CIE Yuv (1960) or CIE Lu'v' (1976) in another variant, lightness can be represented in relation to the coordinates:

\[
\begin{align*}
L' &= \frac{4 \cdot X}{X + 15 \cdot Y + 3 \cdot Z} = \frac{4 \cdot x}{-2 \cdot x + 12 \cdot y + 3} \\
v' &= \frac{9 \cdot Y}{X + 15 \cdot Y + 3 \cdot Z} = \frac{9 \cdot y}{-2 \cdot x + 12 \cdot y + 3}
\end{align*}
\]

for the Lu'v' space and the reverse transformation:

\[
\begin{align*}
x &= \frac{27 \cdot u'}{18 \cdot u' - 48 \cdot v' + 36} = \frac{9 \cdot u'}{6 \cdot u' - 16 \cdot v' + 12} \\
v &= \frac{12 \cdot v'}{18 \cdot u' - 48 \cdot v' + 36} = \frac{4 \cdot v'}{6 \cdot u' - 16 \cdot v' + 12}
\end{align*}
\]

where \(x\) and \(y\) are the values in the CIE xy – 2° space, illuminant C with values: \(u'_0 = 0.2009\) and \(v'_0 = 0.4610\).

The CIE L*u*v* is appropriate for characterising the coloured light of monitors, TVs etc.

![Figure 6: CIE L*u*v*](image)

For the CIE L* u* v* space:

\[
\begin{align*}
L' &= \frac{29}{3} \cdot \frac{Y}{Y_0}, \text{ for } \frac{Y}{Y_0} \leq \left(\frac{6}{29}\right) \\
116 \cdot \left(\frac{Y}{Y_0}\right)^{-16}, \text{ for } \frac{Y}{Y_0} > \left(\frac{6}{29}\right)
\end{align*}
\]

\[
\begin{align*}
u' &= 13 \cdot L' \cdot (u' - u'_0) \\
v' &= 13 \cdot L' \cdot (v' - v'_0)
\end{align*}
\]

The reverse relations:

\[
\begin{align*}
u &= \frac{u^*}{13} \pm u'_0, \\
v &= \frac{v^*}{13} \pm v'_0.
\end{align*}
\]
the values in the CIE u'v' colour space.

\[
Y = \begin{cases} 
Y_u \cdot L' \left( \frac{3}{29} \right), & \text{for } L' \leq 8 \\
Y_v \left( \frac{L'+16}{116} \right), & \text{for } L' > 8 
\end{cases}
\]

\[
X = Y_u \cdot \frac{9 \cdot u'}{4 \cdot v'}, \\
Z = Y_v \cdot \frac{12 - 3 \cdot u' - 20 \cdot v'}{4 \cdot v'}, \text{ for } L' < 8
\]

and

\[
X = -9 \cdot Y_u \cdot \frac{u'}{(u'-4 \cdot v' - u'v')}, \\
Z = (9 \cdot Y - 15 \cdot v'Y - v'X)/(3 \cdot v'), \text{ for } L' > 8.
\]

X, Y, Z in the CIE XYZ space.

The cylindrical representation version includes the elements:

\[
C_{ab}^* = \sqrt{[a^*]^2 + [b^*]^2}, \\
h_{ab}^* = \tan^{-1} \left( \frac{b^*}{a^*} \right).
\]

The L*a*b* diagram may be used concurrently with the L* u' v' diagram.

The graph of the CIE L*a*b* to CIE L*chuv transfer with L* = constant.

CIE L*chuv

L*chuv represents the cylindrical version of the CIE L’u’v’ space where C*uv stands for the chroma, while h*uv stands for saturation.

\[
C_{ch}^* = \sqrt{[u']^2 + [v']^2}, \\
h_{ch}^* = \tan^{-1} \left( \frac{v'}{u'} \right).
\]

On the other hand, the correlation of saturation may be defined through the formula:

\[
s_{uv} = \frac{C^*}{L} = 13 \cdot \sqrt{(u' - v_0)^2 + (v' - v_0)^2}.
\]

CIE L*a*b*, L*u*v* and L*c*h* are non-linear functions in relation to XYZ. The L* components is linked to lightness perception. The other two components describe the chroma (C*a b or C*uv).

To calculate the differences the formula below is used:

\[
\Delta H_{ab}^* = \sqrt{(\Delta L_{ab}^*)^2 + (\Delta C_{ab}^*)^2}; \\
\Delta H_{ab}^* = 2 \cdot \sqrt{C_{ab,1}^* \cdot C_{ab,2}^* \cdot \sin \left( \frac{\Delta h_{ab}}{2} \right)} \text{, } h_{ab} < 0 \text{ or } > 0.
\]

**Colour Difference**

**CIE ΔEab 1976**

The L*, a*, b* values are employed in calculating colour difference:

\[
\Delta E_{ab,76}^* = \sqrt{(\Delta L)^2 + (\Delta a^*)^2 + (\Delta b^*)^2},
\]

\[
\Delta L' = L'_1 - L'_2, \\
\Delta a^* = a'_1 - a'_2, \\
\Delta b^* = b'_1 - b'_2.
\]

The value is depended upon the illuminant employed and the standard (CIE 1931 - 2° variant or the CIE 1964 - 10° variant).

**CIE ΔEuv 1976**

\[
\Delta E_{uv,76}^* = \sqrt{(\Delta u')^2 + (\Delta v')^2}.
\]

\[
\Delta L' = L'_1 - L'_2, \\
\Delta u' = u'_1 - u'_2, \\
\Delta v' = v'_1 - v'_2.
\]

**CIE ΔEch 1976**

\[
\Delta E_{ch}^* = \sqrt{(\Delta C)^2 + (\Delta H)^2},
\]

\[
\Delta L' = L'_1 - L'_2, \\
\Delta C' = C'_1 - C'_2, \\
\Delta H' = H'_1 - H'_2.
\]

**Figure 7:** The graph of the CIE L*a*b* to CIE L*chuv transfer with L* = constant.

**Figure 8:** CIE L*ch space

**Figure 9:** CIE ΔEab 1976 calculations

**Figure 10:** CIE ΔEch 1976 calculations
If $\Delta L^*$, $\Delta C^*$, $\Delta H^*$ are not equal following regular observation, the following adjustments are performed:

$$\Delta E_{hc}^* = \left( \frac{\Delta L^*}{l} \right)^2 + \left( \frac{\Delta C^*}{I} \right)^2 + \left( \frac{\Delta H^*}{h} \right)^2.$$  

**CIE $\Delta E_{ab}$ 1994**

For minor colour differences the formula below applies:

$$\Delta E_{ab,94}^* = \left( \frac{\Delta L^*}{K_L \cdot S_L} \right)^2 + \left( \frac{\Delta C^*}{K_C \cdot S_C} \right)^2 + \left( \frac{\Delta H^*}{K_H \cdot S_H} \right)^2,$$

$S_L = 1$

$S_C = 1 + 0.045 \cdot C^*$

$S_H = 1 + 0.015 \cdot C^*.$

The weight functions $S_L$, $S_C$, $S_H$ are dependent upon the position in the CIE $L^* a^* b^*$ space.

The parametric factors $K_L$, $K_C$, $K_H$ with an approximate value of 1 are dependent upon on the criteria of perceptibility (texture, background, etc.) or acceptability and change with light, chroma or saturation in various industrial applications.

**CIE $\Delta E_{ab}$ 2000**

The formula is derived from the CIE $\Delta E_{ab}$, 94 equation.

$$\Delta E_{ab,00}^{1.2} = \Delta E_{ab,00}^{1.2,00} \left( L'_L, a'_L, b'_L; L'_2, a'_2, b'_2 \right)$$

and observes the symmetry condition:

$$\Delta E_{ab,00}^{1.2,00} \left( L'_L, a'_L, b'_L; L'_2, a'_2, b'_2 \right) = \Delta E_{ab,00}^{1.2,01} \left( L'_2, a'_2, b'_2; L'_L, a'_L, b'_L \right)$$

$$\Delta E_{ab,00}^{1.2} = \left( \frac{\Delta L^*}{K_L \cdot S_L} \right)^2 + \left( \frac{\Delta C^*}{K_C \cdot S_C} \right)^2 + \left( \frac{\Delta H^*}{K_H \cdot S_H} \right)^2 + \Delta R,$$

$$\Delta R = R_T \cdot f(\Delta C', \Delta H') = R_T \left( \frac{\Delta C'}{K_L \cdot S_L} \right) \left( \frac{\Delta H'}{K_H \cdot S_H} \right) \Delta$$


Figure 11: Calculation of the average hue

**Stages of calculation**

1. $C_i$, $h_i$ are calculated:

$$C_{i,ab}^* = \sqrt{\left( a_i^* \right)^2 + \left( b_i^* \right)^2}, \quad i = 1, 2$$

$$C_{ab}^* = \frac{C_{i,ab}^* + C_{2,ab}^*}{2}$$

$$G = 0.5 \cdot \begin{pmatrix} - \frac{C_{ab}^*}{1 + 25^*} \end{pmatrix}$$

$$\hat{a}_i = (1 + G) \cdot a_i^*, \quad i = 1, 2$$

$$C_i = \sqrt{(\hat{a}_i^*)^2 + (\hat{b}_i^*)^2}, \quad i = 1, 2$$

$$h_i = \begin{cases} 0, & a_i = 0, \quad i = 1, 2 \\ \tan^{-1}(\hat{b}_i^*, \hat{a}_i^*), & \hat{b}_i^* \neq 0, \hat{a}_i^* > 0 \end{cases}$$

- $h_i$ is converted from radians to degrees by multiplying the latter by $180/\pi$.

2. The following calculations are performed:

$$\Delta L = L_E - L_i$$

$$\Delta C' = C'_i - C_i$$

$$h = \begin{cases} 0, & C_i, C'_i = 0, \quad |h_i - h'_i| \leq 180^\circ \\ \left( h_i - h'_i \right) - 360, & C_i, C'_i \neq 0, \quad |h_i - h'_i| > 180^\circ \\ \left( h_i - h'_i \right) + 360, & C_i, C'_i \neq 0, \quad |h_i - h'_i| < -180^\circ \end{cases}$$

$$\Delta h' = 2 \cdot \sqrt{C_i^* \cdot C_i} \cdot \sin \left( \frac{\Delta h}{2} \right)$$

and $\Delta H'$ are not absolute values and influence $\Delta R$

3. **CIE $\Delta E_{ab}$ 2000** is calculated:

$$\tilde{L} = \left( L_i + L'_E \right) \frac{2}{C_i + C'_i}$$

$$\tilde{C} = \left( C_i + C'_i \right) \frac{2}{C_i \cdot C'_i}$$

$$h' = \begin{cases} \left( h_i + h'_i \right) - 360, & C_i, C'_i \neq 0, \quad |h_i - h'_i| > 180^\circ; \left( h_i + h'_i \right) \geq 360 \\ \left( h_i + h'_i \right) + 360, & C_i, C'_i \neq 0, \quad |h_i - h'_i| < -180^\circ; \left( h_i + h'_i \right) < 360 \end{cases}$$

$$T = 1 - 0.17 \cdot \cos \left( h' - 30 \right) + 0.24 \cdot \cos \left( 2 \cdot h' \right) + 0.32 \cdot \cos \left( 3 \cdot h' + 60 \right) - 0.2 \cdot \cos \left( 4 \cdot h' - 63 \right)$$

$$\Delta \theta = 30 \cdot \exp \left[ \frac{\left( h' - 275^\circ \right)^2}{25} \right]$$

$$R_E = 2 \cdot \frac{\left( C_i^* \right)^2}{C_i^* + 25^*}$$
$S_c = 1 + 0.045 \cdot C''$

$S_H = 1 + 0.015 \cdot C'' \cdot T$

$R_T = -\sin(2 \cdot \Delta \theta) \cdot R_c$

$\Delta E_{ab,00} = \sqrt{\frac{\Delta L^*}{K_c \cdot S_c} + \frac{\Delta C^*}{K_c \cdot S_c} + \frac{\Delta H^*}{K_h \cdot S_h}} + \Delta R$

$\Delta R = R_T \cdot f(\Delta C', \Delta H') = R_T \cdot \left( \frac{\Delta C'}{k_1 \cdot S_c} \right) \frac{\Delta H'}{k_h \cdot S_h} P_0$

CIE 1931 2°  CIE 1964 10°

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x'0</th>
<th>y'0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31271</td>
<td>0.32902</td>
<td>0.31382</td>
<td>0.33100</td>
</tr>
<tr>
<td>0.1987</td>
<td>0.4683</td>
<td>0.1979</td>
<td>0.4696</td>
</tr>
<tr>
<td>23,482</td>
<td>37,384</td>
<td>60,1046</td>
<td></td>
</tr>
</tbody>
</table>

Hunter $\Delta E$

$\Delta L = L_{\text{sample}} - L_{\text{standard}}$

$\Delta a = a_{\text{sample}} - a_{\text{standard}}$

$\Delta b = b_{\text{sample}} - b_{\text{standard}}$

$\Delta E = \sqrt{\Delta L^2 + \Delta a^2 + \Delta b^2}$

Values $\Delta L$, $\Delta a$, $\Delta b$ are positive provided the sample is more brighter, redder, and more yellow respectively than the standard sample; same values are negative if the sample is darker, greener, or bluer, respectively, than the standard sample.


Figure 12: WA.jpg

RGB =

255 255 253
136 253 0
1 81 254
188 40 255
254 0 43
254 186 185
255 134 185
255 81 136
254 42 82
254 0 39
0 0 0
82 1 0
108 0 0
186 0 41

LAB =

235.3109 -39.3068 198.3245
209.8587 -63.2874 177.0735
42.5157 41.5979 -0.0397
56.4077 54.5670 14.4767
Colour difference in relation to D65

WA image
345.0946  230.2285  179.7420
322.5115  210.0017  186.2778

WD image (corrected WA image)
342.8389  229.5993  179.3253
318.0038  205.9812  186.5974

CONCLUSIONS

Based on a complex research of state-of-the-art specialist literature (references 1-13), the paper provides the condensed CIE formulas for assessing colour, the underlying mathematical reasoning and graphic representation methods commonly used in practice.

The representation of colour in modern spaces - CIE L’a b’, CIE L’u v’ - aims to deliver uniform distribution.

Differences in colour rendered by the ever more complex CIE 1976, 1994, 2000 variants encompass a, b and c, h chromaticity as axes and L as the main axis.

The mathematical calculation algorithm is integrated using particular functions in the Matlab environment, which extended to processing trichromatic digital images.

Matlab extensions enable the rapid calculation of specific colour measurements.

In relation to the illuminant D65, differences emerge in the calculation of colour difference, as reflected by the absolute difference and relative difference.

Through correction (WD image), colour difference may be amplified or diminished.

The CIE 2000 calculation alternative, for image variations WA and WD, delivers values in a more limited range.

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