METHODS MATHEMATICAL PROOF. DEDUCTIVE METHOD.

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Abstract

The article presents the deductive method of demonstration followed by some representative examples. If in Mathematics we can choose the best demonstration when we teach it we have to adapt ourselves to the most convenient variants. We analyse problems of convergeney of the following straight line important in tetrahedron, the median line(the segment which unifies the vertex with the centre of the opponent face) the trimedians (a segment which unifies the middle of the two opponent margins) the perpendicular raised on the faces of the tetrahedron in the circumscribed centers. Hypotheses and conclusions, causes, effects, motivations and justifications, extrapolations and generalizations are given in all the domains of human activity. The chain of implication is sure too: form A results B, from B results C and so on and it is true too that the causal effect is not uniquely determined. Having the conclusion of the theorem stated that the chain receives a direction from the hypothesis to conclusion eliminating the significances. The utility of the article is by the great member of adequate examples from the method of teaching Maths problems and the teaching practice examples which offer the undergraduates-a helping hand in their formation.

Keywords: deductive method, logical deduction, median plane, problem situation.

The article offers the conceptual frame and the necessary methodological references for the future high school and secondary Maths teachers'training through the initial formation.

The material is useful too, for those preparing their Master Degree and for those who are at the beginning of their career or those preparing to become teacher in ordinary or those preparing their second degree exam.

The general problems of psycho pedagogy and the ones specific to the process of teaching, learning and assessment in Mathematics are coherently connected being based both on a vast documentation and on the author's didactic experience. Starting from the theorem of the median line in a triangle the problem of concurrence of the following important lines in a tetrahedron is analysed: the median lines (the segment unifying the vertex with the central point of the opponent surface), the bimedians (the segments unifying the middle of two opponent margins) the perpendiculars raised in their circumscribed centres.

MATERIAL AND METHOD

The *deductive method* is specific to Mathematics in its most pure form. We do not claim that other curricular subjects do not use *logical deduction*.

Hypotheses and conclusions, causes and effects, motivations and justifications, extrapolations and generalizations are given in all branches of human activities but in Maths hypotheses are sure, true even if sometimes on the truthfulness of a hypothesis there is a requirement as for example a periodical function or we are given an equilateral triangle or if A= 90° then. We even can accept things which can not be achieved like in the method of reduction ad absurdum in which the beginning is given by: let us suppose that (supposition which usually contradicts common sense). The chain of implication is sure too from A results B, from B results C and so on but it is also true that the causal chain is not uniquely determined. But having the conclusion of the theorem stated, the chain receives a direction from the hypothesis to conclusion eliminating its significancies. Nevertheless its uniqueness is not assured. What does another demonstration signify for this theorem?

We do not take into consideration to lengthen or to shorten the initial demonstration. but to different demonstrations as idea or language(the synthetic, analytical, vectorial geometry or complex numbers).

If in Maths we can choose between the best demonstration in teaching we must adopt the

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most convenient variants. The modernisation of Maths teaching was realized both through its content and the use of more *modern* means of demonstration, taking into account of course age particularities.

A good example is the use of vectorial calculations (modern of over a century). Of course we could have objections to the replacement of more elementary means with some more elaborate ones contradicting this way the principle of obeying age particularities which is true only partially.

We can illustrate the above statement with the following examples.

The identity (x-a)(b+c)+(x-b)+(x-b)(c-a)+(x-c)(a-b)=0 is valid in R or C. Let us take into consideration a, b, c, x $\in$C as affixes of the points A,B,C, P. We have successively:

$(x - a)(b - c) = -(x - b)(c - a) - (x - c)(a - b) \Rightarrow |(x - a)(b - c)| = |-(x - b)(c - a) - (x - c)(a - b)| \Rightarrow |x - a|\,|b - c| \le |x - b|\,|c - a| + |x - c|\,|a - b|$, if ABC is an equilateral triangle (then |b-c|=|c-a|=|a-b|=0 and after simplification we obtain the inequality PA $\le$ PB + PC which together with two analogous ones gives us *Pompeius' theorem* (we accept P on the circumscribed circle in which case the triangle which can be formed with PA,PB,PC is a degenerated one).

If the triangle ABC is not an equilateral one we obtain a generalization of the theorem (with the following sizes BA, PC, AC, PB, CB, BA, considered as lengths we can build a triangle).

If we interpret vectorially the initial relations and the product is scalary we have:

$\overline{PA}\cdot\overline{BC}+\overline{PB}\cdot\overline{CA}+\overline{PC}\cdot\overline{AB}=\overline{0}$. If A, B, C, P are coplanary and then

$\overline{PA}\perp\overline{PB}, \overline{PB}\perp\overline{PC}\Rightarrow\overline{PC}\perp\overline{AB}$ it was demonstated that the three heights of the triangle ABC are concurrent in the point P.

If the points are in space ,the result is the demonstration of a known problem :in any tetrahedron PABC, if PA $\perp$ BC and PB $\perp$ CA then PC $\perp$ AB.

The above examples and others imply a methodological question: do we have to obey the demonstration in the text book? Now when we use alternative textbooks the challenge is bigger because once you recommended a text book it is understood that the teacher adheres to the author's opinion.

We consider that the most suitable answer is in many cases the realization of the most useful compromise.

The alternative text book asks the teacher to find other examples, other solutions and demonstrations another order of the theorems.

For those who frequently look for other demonstrations, we appreciate that the best is to present too the variant in the textbook and draw a comparison between the two ways. A classical example is the taking away of the concurrency of the medians, of the bisecting lines and the heights of the triangle using Ceva's theorem. The unification of three separate demonstrations has a special impact on pupils.

For example after giving the definition of the median line. In a triangle, the segment which unifies the middles of two sides is called a *median line*, we demonstrate the theorem of the median line in a triangle. *In a triangle the segment which unifies the middle of two sides is parallel to the third one and its length equals half of the third one's length*. (Achiri I. 2002).

Hypothesis: ABC is a triangle

Conclusion: a) MN//BC, $[AM]=[MB]$, $[AN]=[NC]$; b) MN=BC/2

Demonstration:

a) We see that $\left.\begin{matrix} AM/MB=1 \\ AN/NC=1 \end{matrix}\right\}$

we deduce that AM/MB=AN/NC.

In accordance with the reciprocal theorem of Thales we deduce MN//BC.

b) Be $[BP]\equiv[PC]$, $[NP]$ median line we deduce in accordance with a) NP//AB. We deduce NP//MB (1). Because MN//BC we deduce MN//BP (2). From (1) and (2) we deduce BMNP is a parallelogram where from results [MN]=[BP]. But [BP]=BC/2 where we deduce that MN=BC/2 too.

This an example of deductive demonstration of some problems of geometry in space. We will analyse the following important lines in a tetrahedron: the median (the segment which unifies the vertex to the weight centre of the opponent surface).

The bimedians (the segment which unifies the middle of the two opponent edges), the perpendicular raised on the faces of the tetrahedron in their subscribed centres.

There appeared the *following problems:*

i) In a tetrahedron are the heights concurrent?

ii) Is there a condition if and only if for a height in a tetrahedron to be concurrent?

iii) Give an example.

Problem1 Be M the middle of the edge CD of a tetrahedron ABCD, I$\in$BM and J$\in$AM the points of the weight centres of the faces BCD and ACD. Demonstrate that the median AI and BJ are concurrent. *(fig. 1)*.

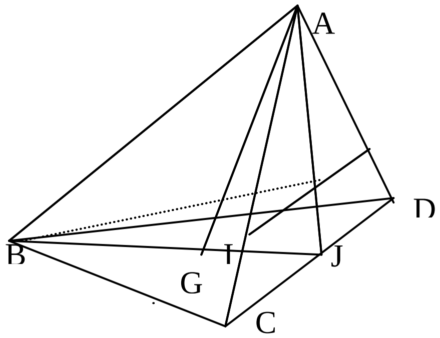


fig. 1

The medians of the tetrahedron being coplanar (are in he same plan) two by two concurrent thus they all have a common point G which divides each median in the ratio1/3 and it is called the weight centre of the tetrahedron. As far as the concurrence of the bimedians, it is demonstrated that:

In a tetrahedron ABCD the bimedians MB, NQ, and RS are concurrent?
($M \in BC$, $MB \equiv MC$; $N \in CD$, $NC \equiv ND$; $P \in AD$, $PA \equiv PD$; $Q \in AB$, $QA \equiv QB$; $R \in AC$, $RA \equiv RC$; $S \in BD$, $SA \equiv SO$; (*fig. 2*).

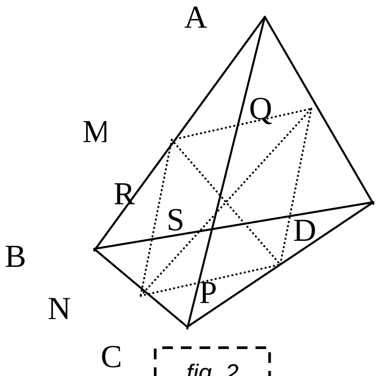


fig. 2

For the demonstration we can see that the quadrilateral MNPQ is a parallelogram and QN and MP are concurrent, being diagonals in this parallelogram dividing it into halves. In an analogous way RS passes through the middle of QN and MP. We can demonstrate that the point of concurrence of the medians is the weight centre G too because the bimedians MP, for example is the median of the median plan (MAD) and of the median plan (NBD) and because the weight centre belongs to each of the median plans of the tetrahedron results that G belongs to each median line.

As far as the concurrence of the perpendicular lines raised in the centre of the circumscribed circles of the lateral faces we will solve first the fact that the mediator plans of the sides of a triangle have a common straight line. (*fig.3*).

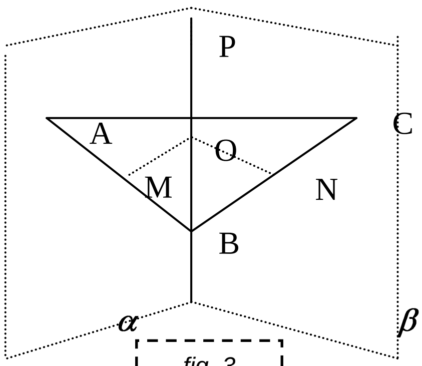


fig. 3

Building the mediator plans α and β of the sides AB ,BC from the triangle ABC we will have $\alpha \cap \beta$ = PO and $PA \equiv PB \equiv PC$.

The straight line PO of the intersection of the plan perpendicular on the mediators of the triangle ABC is called mediator in space of the triangle and any point is at a equal distance from the vertex of the triangle.

Then we demonstrate the fact "the six mediator plans perpendicular on the sides of the tetrahedron are concurrent in a point which is at a equal distance from the vertex of a tetrahedron" *(fig. 4).*

IF m_1 is the mediator in space of the triangle ABC and μ the mediator plan of the side AD where $O = m_1 \cap \mu$ we have $OA \equiv OB \equiv OC \equiv OD$ = R meaning that O belongs to other perpendicular plans.

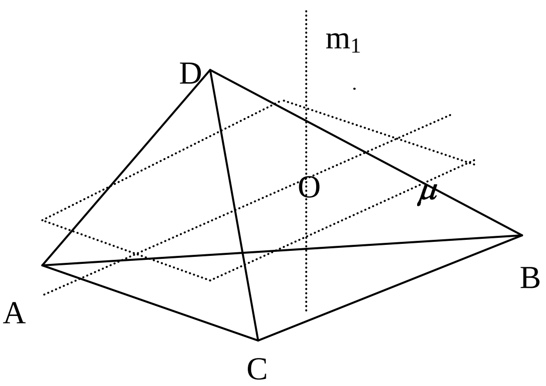


fig. 4

i) As far as the heights of a tetrahedron must be concurrent we will define the tetrahedron first as an orthocentric or orthogonal (denomination introduced by Steiner 1927) that is a tetrahedron is orthocentric if it has the opposed hedges perpendicular. The centre Sphere O and radius is called circumscribed sphere of a tetrahedron.

ii) Are the heights concurrent? The answer is no. We can justify this through an example: In the tetrahedron DABC with the base Δ ABC equilateral and DA $\perp$ (ABC), the heights BM, CN, DA are not concurrent.

Lemma:

If in tetrahedron ABCD two opposed plans of opponent sides are perpendiculars then the other two sides are perpendicular.(8. Pogorelov A, V, 1991). *(fig.6)*.

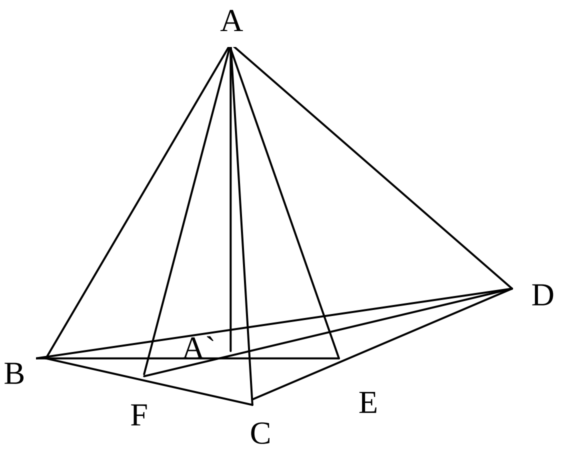


fig. 5

Demonstration:

Be AB ⊥ CD and BC ⊥ AD, we deduce AE ⊥ DC and AF ⊥ BC. It results CD⊥(ABE) and BC ⊥ (ADF).

If AA′ = (ABE) ⋂ (ADF) there AA′⊥ (BCD). But CA′⊥BD and there BD⊥(ACA′) results BD ⊥ AC.

We demonstrate the theorems

Theorem 1: The heights in a tetrahedron are concurrent if and only if the tetrahedron is orthocentric.

Demonstration: We demonstrate that the heights A,D are concurrent when AD⊥BC, (L'Hulier 1782). Be A' and D′ the orthogonal projections of point A and D in the plans (DBC) and (ABC) and AA' with concurrent DD'. AA′⊥ (DBC) ⇒ AA′⊥BC and DD′⊥(ABC)⇒DD′⊥BC.

In conclusion BC being perpendicular on two straight concurrent lines it will be perpendicular on the plan (AA',DD') and perpendicular on AD too. In a reciprocal way if AD⊥BC, be F the foot of the perpendicular from A on BC. From BC⊥AD and BC⊥AF results BC⊥(AFD)⇒BC⊥AA′. Because AA′⊂(AFD) and DD′⊂(AFD) it results that AA' end DD' are the heights (AFD). AA' and DD' are concurrent then the other two heights BB' and CC' are concurrent. It is then that AA′∩DD′=∅ ⇒ AD ⊥ BC and BB′∩ CC′=∅.

We can demonstrate that the heights AA' and BB' are concurrent in H, and then the common perpendicular of the edges AD and BD contains H, because if the plan (AA', DD') cuts BC in F it results that FH is the third height and FH⊥AD the same way BC⊥(AFD) ⇒ FH⊥BC.

Analogically we demonstrate that if the heights AA' and DD' are cut in H and BB' and CC' in H′≠ H then HH' is the common perpendicular of the opponent edges AD and BC.

Of the 3 conditions of perpendicularity of orthocentric tetrahedron results the concurrence of the heights. Another theorem particularly interesting to be analysed is:

Theorem 2: A tetrahedron is orthocentric if and only if: $AB^2+CD^2 =$ $AC^2+BD^2=AD^2+BC^2$.

Demonstration: We must demonstrate that the heights in A and D of the tetrahedron ABCD are secant if and only if $AB^2+CD^2=AC^2+BD^2$. Be F and F_1 the orthogonal projections of the points A and D on BC. We have the following relations:

$$AB^2-AC^2=(AF^2+BF^2)-(AF^2+FC^2)=$$

$$BF^2-FC^2=(BF+FC)(BF-FC)=$$

$$BC(BF-FC).$$

$$BD^2-CD^2=(DF_1^2+BF_1^2)-(DF_1^2+F_1C^2)=$$

$$BF_1^2-CF_1^2=(BF_1+CF_1)(BF_1-CF_1)=$$

$$BC(BF_1-F_1C).$$

We can see that AA' and DD' are secants if and only if F coincides with F1. From the previous equalities with

$$F=F_1 \Rightarrow AB^2-AC^2=BD^2-CD^2 \Rightarrow$$

$$AB^2+CD^2=BD^2+AC^2$$

Reciprocally

$$AB^2+CD^2=BD^2+AC^2 \Rightarrow din\ (1)$$

$$BC(BF-FC)=BC(BF_1-F_1C)$$

$$BF-FC=(BF+FF_1)-(FF_1+FC) \Rightarrow 2FF_1=0.$$

so F and F1 coincide. demonstrating analogically and taking into account the concurrence of the other heights it results that:

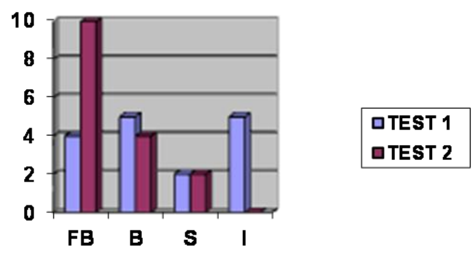
$$AB^2+CD^2=AC^2+BD^2=AD^2+BC^2.$$

Very easily from here we can deduce too that the tetrahedron ABCD is orthocentrical if and only if the 3 bimedians are congruent.

The results and their interpretation

Through the experiment done with the second year Math undergraduates of" VASILE ALECSANDRI" UNIVERSITYon an initial test1 and a final test 2 it was proved that the formation and the development of skills and abilities for solving the exercises and problems which contain elements of geometry in the secondary school are possible if we use various livening methods and procedures.

As a result in the experiment I organized and managed my aim was to investigate the



efficiency of using the deductive method in solving geometry problems in secondary school.

The obtained data proved that different situations created in the experiment represent a valency specific to different categories of undergraduates (very good, good, mediocre).

Confronting students with different situations and solving problems using the logical deduction is a means of discovery, which mobilizes the pupils more. Thus the hypothesis of my work was confirmed and I was able to reveal the existence of the possibility of creating characteristic situations specific to the teaching activities which take in consideration the amplification of inner reasons such as changing some extrinsecal reasons into intrinsic reasons.

At the same time the results of investigation confirm the ***hypothesis*** *that if we use and turn to good account the deductive method in all lesson stages while teaching geometry in the secondary school then all the lessons will be efficient and the results of the pupils better*.

The results obtained through the use of the proofs led to the following findings.

a) The demonstrations and their use belong to the motivational situation being efficient because they mobilize the undergraduates and therefore the students when they teach.

b) The obtained data demonstrated that the results are superior in all tests with deductive demonstrations and use.

c) They activated the undergraduates with poor results too, eliminating their fear, shyness, discouragement.

d) Any notion introduced or consolidated with their help is easier accessible contributing to the formation of abilities and skills for the demonstration of problems through the deductive method.

Using the logical deduction as an active participating method in solving the theorems and problems offer the possibility to the future teachers to know pupils better, to know their individual particularities, their proper style, intelligence, will, temperament, behaviour, in short their personality.

I consider that the proposed objective and the hypothesis of my research were confirmed and the importance of the teaching- learning elements of geometry through the deductive method is one of the must efficient methods of demonstrating and solving theoretical and practical problems.

My strategy of presenting the way the undergraduates took part into solving the tasks is an attempt to use the theoretical and practical knowledge from works of speciality combined with my experience.

CONCLUSIONS

The article offers the conceptual frame and the necessary methodological references for the future high school and secondary Maths teachers' training through the initial formation.

The material is useful too, for those preparing their Master Degree and for those who are at the beginning of their career or those preparing to become teacher in ordinary
or those preparing their second degree exam.

The general problems of psycho pedagogy and the ones specific to the process of teaching, learning and assessment in Mathematics are coherently connected being based both on a vast documentation and on the author's didactic experience.

The utility of the article is increased by the great number of adequate examples from the method of solving Maths problems and from the educational practice, examples which offer to the undergraduates help for their formation.

The article presents in a general way the deductive method of Maths demonstration after which some representative examples are offered.

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