CHAOS SYNCHRONIZATION IN THREE-DIMENSIONAL SYSTEMS INVOLVED IN WEATHER PREDICTION

Servilia OANCEA[1], Andrei Victor OANCEA[2]

E-mail:lioancea@uaiasi.ro

Abstract

During recent decade a growing interest was observed in the problem of chaotic systems. The idea of synchronizing two identical real autonomous chaotic systems with different initial conditions was first introduced by Pecora and Carroll. Different types of synchronization have been developed in a variety of chaotic systems, such as complete synchronization, phase synchronization, generalized synchronization and so on. In this work a simple feed-back method of control is used to study synchronization for two three-dimensional chaotic systems. A comparison between Lorenz, Lü and Ţigan systems is also presented. The method offers a precise coupling for two identical oscillators. Our results show that the transient time until synchronization depends on initial conditions of two systems and on the control strength. The synchronization is fast when all control strengths were applied to synchronize the two identical three-dimensional systems and the initial conditions of the two systems are nearly. The graphics of MATLAB soft is used to present the synchronization of these chaotic systems.

Key words: Lorenz system, feed-back method, MATLAB

The Lorenz model introduced in 1963 and derived by Lorenz from the equations of convection in the atmosphere has important implications for weather prediction and climate. Lorenz studied thermal variations in the air cell underneath a thunderstorm. He observed the sensitivity to initial conditions, which is the a characteristic of chaos. Now this model is an explicit statement that the atmosphere may exhibit a variety of quasi-periodic regimes that are, although fully deterministic, can produce chaos. During recent decade a growing interest was observed in the problem of chaotic systems and Lorenz system has become one of the most widely studied of ODEs because of its wide range of behaviors.

The idea of synchronizing two identical real autonomous chaotic systems with different initial conditions was first introduced by Pecora and Carroll. Since Pecora and Carroll showed that it is possible to realize chaos synchronization, the behavior of dynamical systems has attracted considerable attention and has been found some potential applications in secure communication, ecological systems, modeling brain activity, chemical reactions and so on (Grzybowski, J.M.V.et al., 2009). Then chaos synchronization was a most studied phenomenon which has been investigated in the literature. Different types of synchronization have been developed in a variety of chaotic systems, such as complete synchronization, phase synchronization, generalized synchronization and so on Grosu, I., 1997, Lei, Y., Xu W., J., 2007, Sun, Met al., 2007, Wu, Y. et al., 2009). We also performed some method of synchronization (Lerescu, A.I. et al., 2004; Lerescu, A.I et al., 2006; Oancea, S.et al., 2009). In this work a simple feed-back method of control is used to study synchronization for some three-dimensional chaotic systems: Lorenz system, Tigan system and Lü system.

MATERIAL AND METHOD

To synchronize two identical chaotic systems we used a simple method for chaos synchronization proposed in [3], [4].

If the chaotic system (master) is:

$\dot{x} = f(x)$ where

$x = (x_1, x_2, ...x_n) \in R_n$

$f(x) = (f_1(x), f_2(x).....f_n(x)) : R^n \to R^n$

then the slave system is:

$\dot{y} = f(y) + \varepsilon(y - x)$

where the functions $\dot{\varepsilon}_i = -\lambda_i (y_i - x_i)^2$ and λ_i are positive constants

Lorenz proposed and studied a 3D autonomous system with nonlinearities, described by:

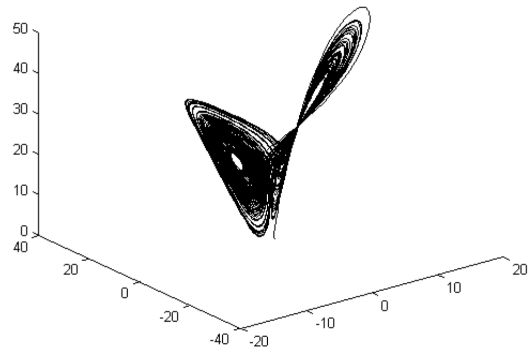
[1] University of Agricultural Sciences and Veterinary Medicine, Biophysics Department, Iasi

[2] Al.I.Cuza" University Iasi

$$\dot{x} = a(y - x)$$
$$\dot{y} = bx - xz - y \quad (1)$$
$$\dot{z} = -cz + xy$$

Here x,y,z are the state variables and a, b, c are positive real constants.

For a = 10, b = 28, c = 8/3 the system has chaotic behaviour. For initial conditions x_0=1, y_0=1, z_0=1 the strange attractor is given in figure 1.



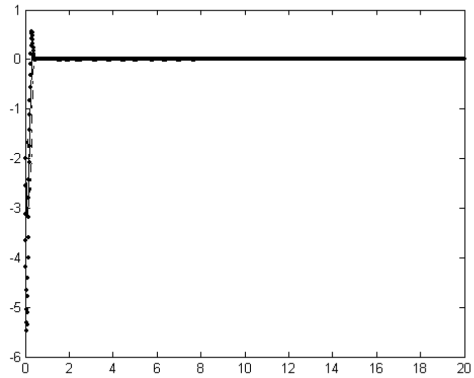


Figure 3 **Synchronization errors between master and slave systems [x(0) = y(0) = z(0) = 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= - 1; ε_1 (0)= ε_2 (0)= ε_3 (0)= 1**

Figure 1 **Phase portrait for Lorenz system**

RESULTS AND DISCUSSIONS

The slave system for the system (1) is:

$$\dot{x} = 10(y_1 - x_1) + \varepsilon_1(x_1 - x)$$
$$\dot{y} = 28x - xz - y + \varepsilon_2(y_1 - y) \quad (2)$$
$$\dot{z} = -8/3z + xy + \varepsilon_3(z_1 - z)$$

The control strength is of the form:

$$\dot{\varepsilon}_1 = -10(x_1 - x)^2$$
$$\dot{\varepsilon}_2 = -10(y_1 - y)^2 \quad (3)$$
$$\dot{\varepsilon}_3 = -10(z_1 - z)^2$$

Fig. 2, 3 and 4 show the syncronization of the two Lorenz systems and figure 5 the control strength.

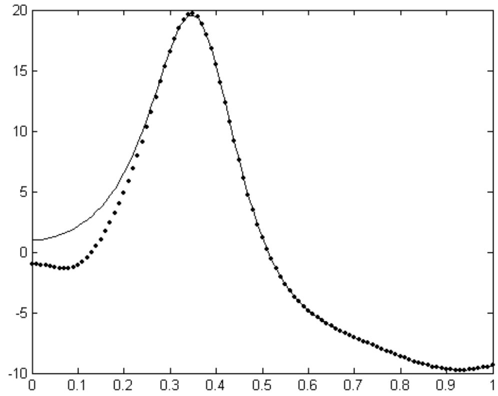


Figure 4 **xt)-, x_1t) ., [x(0) = y(0) = z(0) =1; x_1(0) =y_1(0)= z_1(0) = -1; ε_1 (0)= ε_2 (0)= ε_3 (0)= 1]**

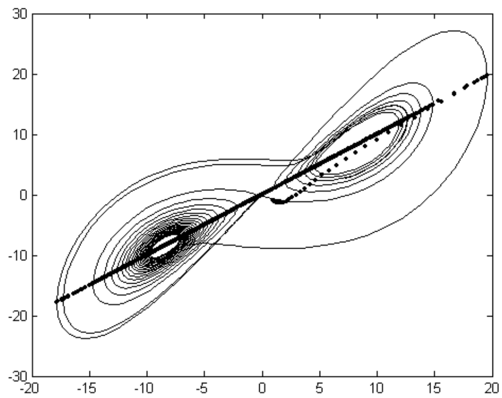


Figure 2 **Phase portrait of (x, y)- and (x, x_1)-., for Lorenz systems with initial conditions [x(0) = y(0) = z(0) = 1; x_1(0) =y_1(0)= z_1(0) =- 1; ε_1 (0)= ε_2 (0)= ε_3 (0) 0)= 1]**

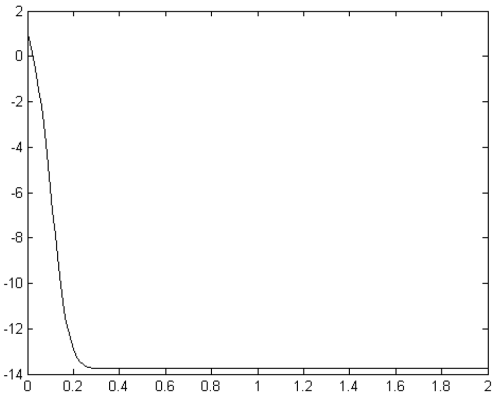


Figure 5 **The control strength ε_1 (t), [x(0) = y(0) = z(0) = 1; x_1(0) =y_1(0)= z_1(0) = - 1; ε_1 (0)= ε_2 (0)= ε_3 (0)= 1]**

Debin Huang (2005), by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows found that the coupling only one variable is sufficient to achieve identical synchronization of a three-dimensional system. When one controler is aplplied in the first equation, we obtained the synchronization for Lorenz system (figure 6).

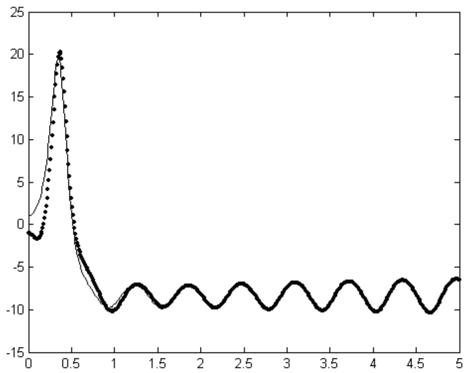


Figure 6 **$x t)$-, $x_1 t)$. [$x(0) = y(0) = z(0) =1$; $x_1(0) = y_1(0) = z_1(0) = -1$; $\varepsilon_1 (0)= 1$]**

From figures 4 and 6 we can see that the synchronization is faster (about three times) when all the contrllers are applied than only one controller have been used to synchronize the two Lorenz systems.

For one controller appled in the second equation the synchronization is also obtained but earlier than in the case before(figure 7).

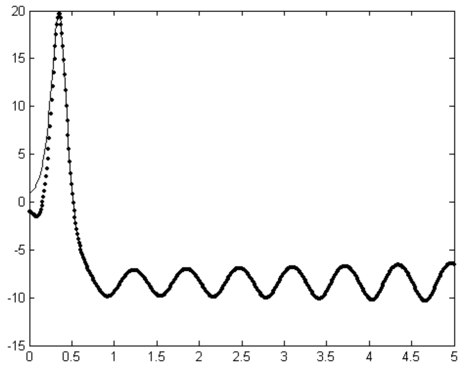


Figure 7 **$x t)$-, $x_1 t)$. [$x(0) = y(0) = z(0) =1$; $x_1(0) = y_1(0) = z_1(0) = -1$; $\varepsilon_2 (0)= 1$]**

When the one controller is applied in the third equation, we obtain the synchronization only the for variable z (figure 8).
For variable x and y we obtain antisynchronization, as the figyres 9 and 10 show.

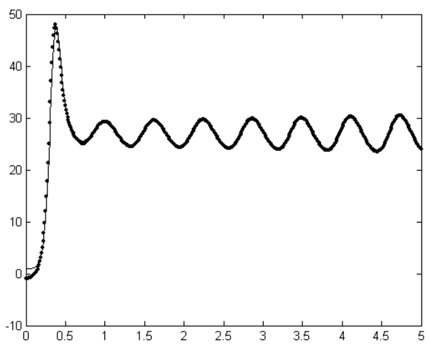


Figure 8 **$z t)$-, $z_1 t)$. [$x(0) = y(0) = z(0) =1$; $x_1(0) = y_1(0) = z_1(0) = -1$; $\varepsilon_3 (0)= 1$]**

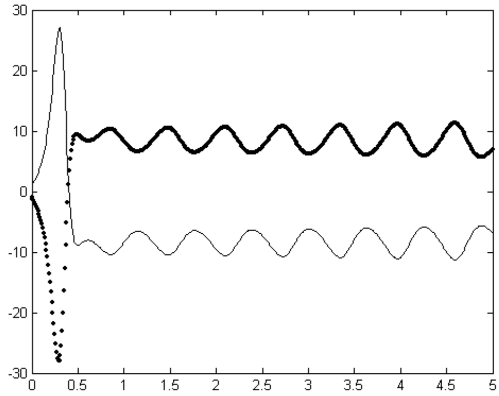


Figure 9 **$y t)$-, $y_1 t)$. [$x(0) = y(0) = z(0) =1$; $x_1(0) = y_1(0) = z_1(0) = -1$; $\varepsilon_3 (0)= 1$]**

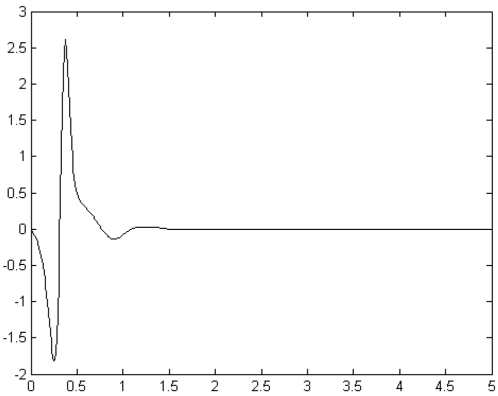


Figure 10 **$y t)$+ $y_1 t)$-, [$x(0) = y(0) = z(0) =1$; $x_1(0) = y_1(0) = z_1(0) = -1$; $\varepsilon_3 (0)= 1$]**

Tigan (2008, 2009) proposed a modified Lorenz system of the form:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (b-a)x - axz \qquad (4) \\ \dot{z} &= -cz + xy \end{aligned}$$

where a=2.1, b=30 and c=0.6. The Tigan atractor is given in figure 11

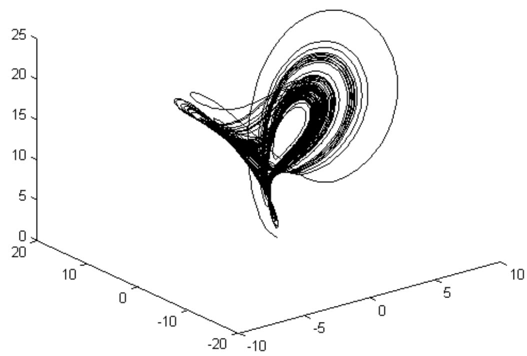


Figure 11 **Phase portrait for Tigan system**

Lü (2005) proposed a 3D system of the form:

$$\dot{x} = a(y - x)$$
$$\dot{y} = bx - xz \qquad (5)$$
$$\dot{z} = -cz + xy$$

where a=30, b=23.2 and c=8.8/3

The atractor for Lü system is:

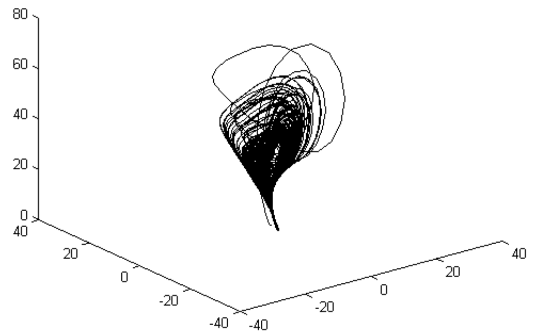


Figure 12 **Phase portrait for Lü system**

When all three controllers have been applied the synchronization of two Tigan systems is fast (figure 13, 14, 15).

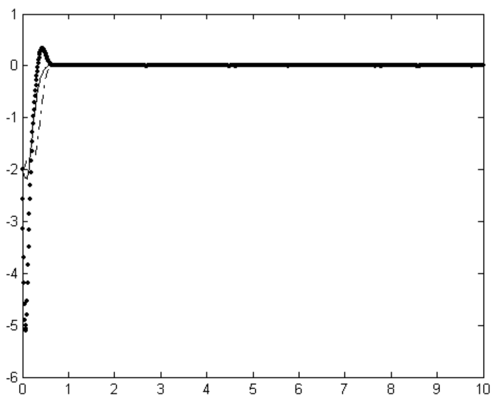


Figure 13 **Synchronization errors between master and slave systems for Tigan system [x(0) = y(0) = z(0) = 1; $x_1(0)$ =$y_1(0)$= $z_1(0)$ = $t_1(0)$= - 1; (0)= ε_2 (0)= ε_3 (0)= 1]**

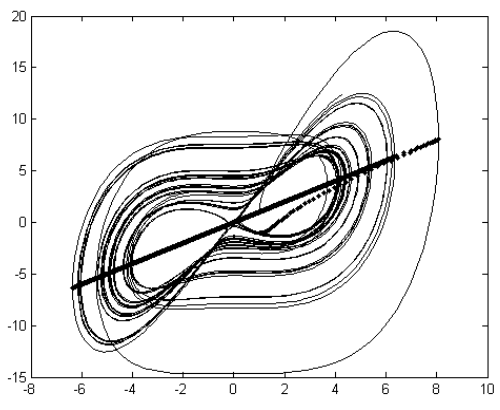


Figure 14 **Phase portrait of (x, y)- and (x, x_1) -., for Tigan systems with initial conditions [x(0) = y(0) = z(0) = 1; $x_1(0)$ =$y_1(0)$= $z_1(0)$ =- 1; ε_1 (0)= ε_2 (0)= ε_3 (0)= 1]**

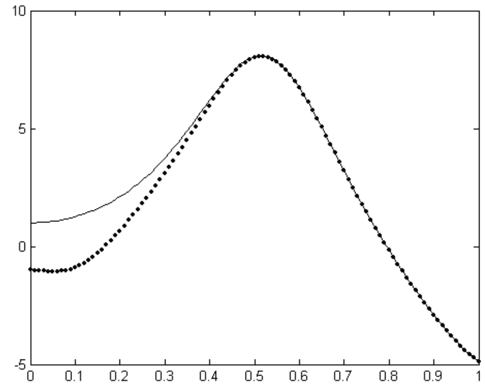


Figure 15 **xt)-, x_1t) ., for Tigan system [x(0) = y(0) = z(0) =1; $x_1(0)$ =$y_1(0)$= $z_1(0)$ = -1; ε_1 (0)= ε_2 (0)= ε_3 (0)= 1]**

For two Lü systems we also obtained the synchronization, when the all controller have been applied (figure 16)

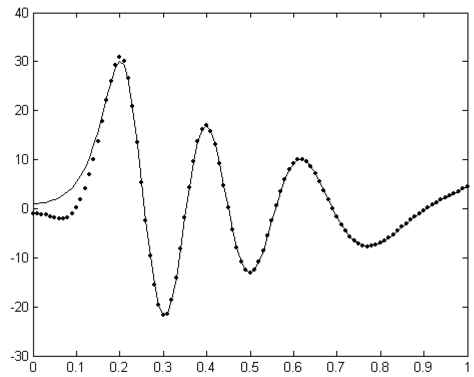


Figure 16 **xt)-, x_1t) ., for Lü system [x(0) = y(0) = z(0) =1; $x_1(0)$ =$y_1(0)$= $z_1(0)$ = -1; ε_1 (0)= ε_2 (0)= ε_3 (0)= 1]**

Figure 17 **xt)-, x_1t) ., for Tigan system [x(0) = y(0) = z(0) =1; $x_1(0)$ =$y_1(0)$= $z_1(0)$ = -1; ε_2 (0)= 1]**

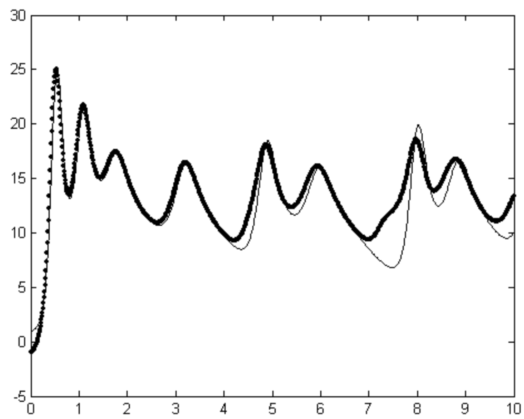
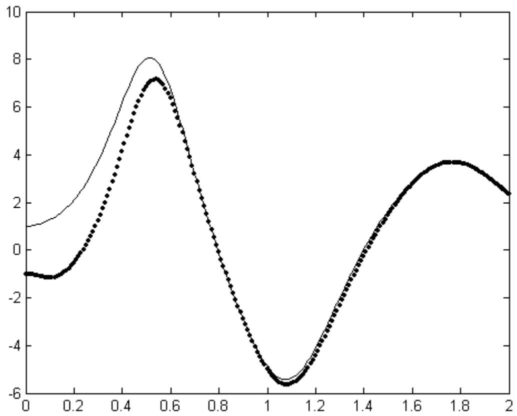


Figure 18 **zt)-, z_1t) ., for Tigan system [x(0) = y(0) = z(0) =1; $x_1(0)$ =$y_1(0)$= $z_1(0)$ = -1; ε_3 (0)= 1]**

A different bahavior is observed for Tigan system and Lü system when only one controller have been applied. If one controler is applied for Tigan system in the first or in the second equation the synchronization is obtained (figure 17). When the controller is applied in the third equation of the system se synchronization is not achieved (figure 18).



For Lü system the synchronization is obtained if one controller is applied only in the second equation.

CONCLUSIONS

In this work the synchronization of two Lorenz system is obtained using a simple feedback method. The syncronization is faster when all three controllers are applied in the slave system than the one controller is applied in the first or in the second equation of the system. In adition we obtain the antisynchronization when the controller is applied in the third equation of the system. By comparison with Lorenz system, Tigan system and Lü system have been synchronized fast enough when three controllers are applied but when one controller is applied it must be applied in the second equation. Therefore for Lorenz system it was easier to synchronize by comparison with th other studied systems, regarding this method of synchronization.

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