THE SYNCHRONIZATION OF TWO FOUR-DIMENSIONAL CHAOTIC SYSTEMS WITH CUBIC NONLINEARITIES

Servilia OANCEA[1], Ioan GROSU[2], Andrei Victor OANCEA[3]

University of Agricultural Sciences and Veterinary Medicine, Iaşi
[2] University of Pharmacy and Medicine, Iaşi
[3]"Al.I.Cuza" University Iaşi

Abstract

The importance of synchronization reveals both in the practical applications that can be obtained and also in the many phenomena that can be explained by synchronization theory. In biology and medicine many systems can be modeled as oscillators or vibratory systems and those systems show a tendency towards synchronous behavior. Since 1990 chaos synchronization has been a topic of great interest as shows the most comprehensive bibliography on chaos control and synchronization. Synchronization is a fundamental process in coupled dynamical systems. This means to design a controller or interconnections that guarantee synchronization of the multi-composed systems with respect to certain desired functional. In this work a simple feed-back method of control is used to study the synchronization for two chaotic four-dimensional systems. The method offers a precise coupling for two identical oscillators. Our results show that the transient time until synchronization depends on initial conditions of two systems and on the control strength. The synchronization is fast (about 3 unities of time) when four control strengths were applied to synchronize the two identical four-dimensional systems with cubic nonlinearities (Qi system), and the initial conditions of the two systems are very closed. The graphics of MATLAB soft is used to present the synchronization of these chaotic systems.

Key words: chaotic systems, Qi system, MATLAB soft

The synchronization phenomenon was first reported by Huygens, who observed that a pair of pendulum clocks hanging from a light weight beam oscillated with the same frequency. Nowadays, there are many papers related with synchronization of rotating bodies and electromechanical systems. As is only natural vibration in machines are usually generated by periodic excitations acting on elastic structure. It is essential to take into account the non-linear coupling, which can exist, between source of the excitatory forces and the vibrating system. The importance of synchronization does not only lie in the practical applications that can be obtained, but also in the many phenomena that be explained by synchronization theory. In biology, medicine and agriculture many systems can be modeled as oscillators or vibratory systems and those systems show a tendency towards synchronous behavior. From the control point of view, the controlled synchronization is the most interesting. That means to design a controller or interconnections that guarantee synchronization of the multi-composed systems with respect to a certain desired functional. Since 1990 chaos synchronization has been a topic of great interest as shows the most comprehensive bibliography on control and synchronization. We also performed some method of synchronization (Grosu, I., 1997; Lerescu, A.I., et al., 2004; 2006; Oancea, Servilia, 2009. In order to formulate the chaos control in this work the synchronization of two four-dimensional systems using a simple feed-back method is presented.

MATERIAL AND METHOD

Hammami, S., 2009; Qi, G. et al, 2005; 2009 proposed and studied a 4D autonomous system with cubic nonlinearities, described by:

$$\begin{aligned} \dot{x} &= a(y-x) + yzt \\ \dot{y} &= b(x+y) - xzt \\ \dot{z} &= -cz + xyt \\ \dot{t} &= -dt + xyz \end{aligned} \qquad (1)$$

Here x,y,z,t are the state variables and a, b, c, d are positive real constants.

For a = 30, b = 10, c = 1, d =10, the systems has chaotic behaviour. For initial conditions x_0=1, y_0=1, z_0=1 and t_0 =1 the strange attractor is given in *fig. 1*.

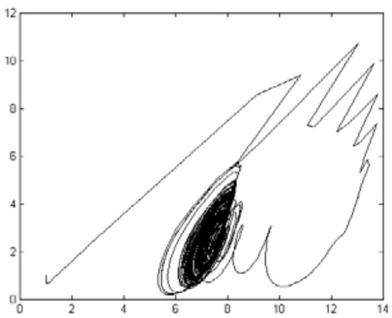

Figure 1 **Phase portrait of (z, t) for Qi four-dimensional system**

To synchronize two Qi systems we used a simple method for chaos synchronization proposed in Guo, R., Li, G., 2009, Guo, W. et al., 2009; Huang, D., 2005.

If the chaotic system (master) is:

$\dot{x} = f(x)$ where $x = (x_1, x_2, ...x_n) \in R_n$

$f(x) = (f_1(x), f_2(x).....f_n(x)) : R^n \to R^n$

then the slave system is:

$\dot{y} = f(y) + \varepsilon(y - x)$

where the functions $\dot{\varepsilon}_i = -\lambda_i (y_i - x_i)^2$

and λ_i are positive constants.

RESULTS AND DISCUSSIONS

The slave system for the system (1) is:

$$
\begin{aligned}
\dot{x}_1 &= 30(y_1 - x_1) + y_1 z_1 t_1 + \varepsilon_1 (x_1 - x) \\
\dot{y}_1 &= 10(x_1 + y_1) - x_1 z_1 t_1 + \varepsilon_2 (y_1 - y) \\
\dot{z}_1 &= -z_1 + x_1 y_1 t_1 + \varepsilon_3 (z_1 - z) \\
\dot{t}_1 &= -10 t_1 + x_1 y_1 z_1 + \varepsilon_3 (t_1 - t)
\end{aligned} \qquad (2)
$$

The control strength is of the form:

$$
\begin{aligned}
\dot{\varepsilon}_1 &= -(x_1 - x)^2 \\
\dot{\varepsilon}_2 &= -(y_1 - y)^2 \qquad (3) \\
\dot{\varepsilon}_3 &= -(z_1 - z)^2 \\
\dot{\varepsilon}_4 &= -(t_1 - t)^2 \qquad (3)
\end{aligned}
$$

Fig. 2, 3, 4 and *5* show the syncronization of the two Qi systems and figure 6 the control strength.

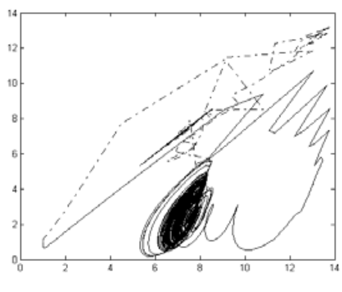

Figure 2 **Phase portrait of (z, t)- and (z, z_1) -. Qi system with initial conditions [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)= ε_2 (0)= ε_3 (0) = ε_4 (0)= 1]**

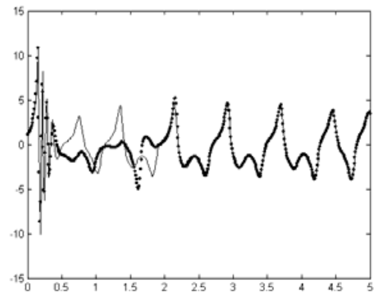

Figure 3 **x(t)-, x_1(t) . [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)= = ε_2 (0)= ε_3 (0) = ε_4 (0)= 1]**

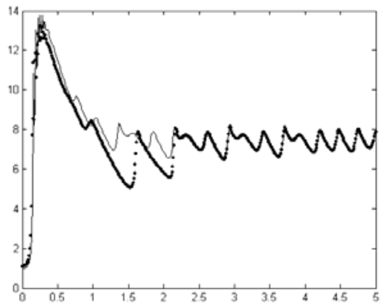

Figure 4 **z(t)-, z_1(t) . [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)= ε_2 (0)= ε_3 (0) = ε_4 (0)=1]**

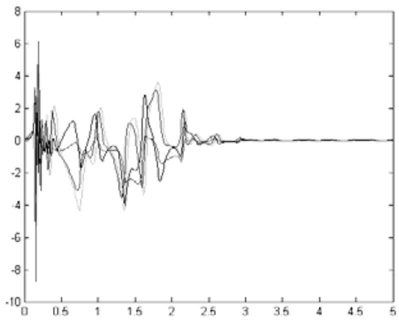

Figure 5 **Synchronization errors between master and slave systems [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)= ε_2 (0)= ε_3 (0) = ε_4 (0)= 1]**

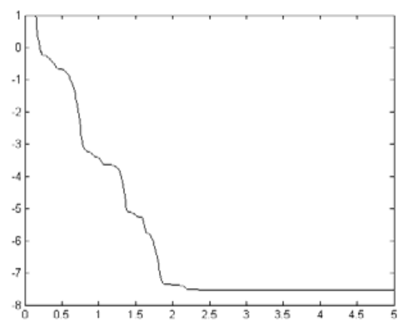

Figure 6 **The control strength ε_1 (t) [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)= ε_2 (0)= ε_3 (0) = ε_4 (0)= 1]**

Debin Huang , 2005, by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows found that the coupling only one variable is sufficient to achieve identical synchronization of a three-dimensional system.

For the system (Grosu I., 1997), we didn't achieve the synchronization if one controller is applied.

For two controllers we obtained the synchronization if the controler is applied only in the first and the second equation of system; then the synchronization is given in *fig. 7, 8*.

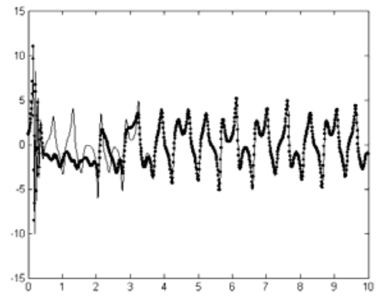

Figure 7 **x(t)-, x_1(t) . [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)=ε_2 (0)= 1]**

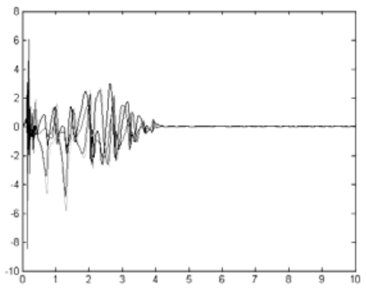

Figure 8 **Synchronization errors between master and slave systems [x(0) = y(0) = z(0) =t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)=ε_2 (0)= 1]**

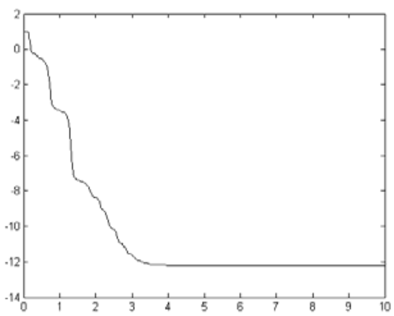

Figure 9 **The control strength ε_1 (t) [x(0) = y(0) = z(0) t_0= 1; x_1(0) =y_1(0)= z_1(0) = t_1(0)= 1.1; ε_1 (0)=ε_2 (0)= 1]**

From *figures 3, 5, 6* we can see that synchronization is fast; this means for about 3 unities of time, when the all controllers are applied. By comparison, from *figures 7, 8, 9* we can see that synchronization is achieved for two controllers two times later.

CONCLUSIONS

In order to formulate the biological control, the synchronization of two Qi four-dimensional systems is presented in this work. The transient time until synchronization depends on initial conditions of two systems and on the control strenght.

The synchronization is fast when four controllers were applied to synchronize the two identical Qi system and the initial conditions are very closed. For these systems we didn't achieve the synchronization if only one controller is applied. For two controllers we obtained the synchronization if the controler is applied only in the first and the second equation of the system. Then the synchronization is about two times later than for all controllers.

The control method described in this paper is very easy and might be useful in the case of the other chaotic systems.

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