USAGE OF MODIFIED DEA MODELS FOR OPTIMIZATION OF THE PRODUCTION STRUCTURE OF AGRICULTURAL ENTERPRISES IN CZECH REPUBLIC

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This contribution is focused on adjustments of Data Envelopment Analysis models in order to improve the production structure of agricultural enterprises respecting their production possibilities and the current market conditions. For evaluation of the enterprises and obtaining recommendations we have chosen Generic Directional Distance Model (GDDM) and Weighted Slack-based measures. The advantage of the mentioned models is a possibility of affecting the recommended changes by weights or directional vectors. In addition, both models can be modified for working with the negative outputs we often meet in the practical use. As this projection on the efficiency frontier is generally non-radial, we propose a method to compute an efficient score too. One of additional objective of this work is to choose the model that gives more realistic recommendation. In the application part, the article is focused on agricultural enterprises farming in a potato growing region, which covers the sizable part of arable land in the Czech Republic.

Key words: *data envelopment analysis, performance, slack-based measure of efficiency, generic directional distance model*

Data envelopment analysis (DEA) is an instrumental to the technical efficiency evaluation of decision making units (DMUs) based on the size of their inputs and their outputs. DEA is used to measure the relative efficiency of comparable units. In particular, while these units use the same kind inputs to produce the same outputs, there may be differences in their performance. The number of units can not be too small, because many units seem to be efficient with a small amount of units in the group and a big criteria number.

One item of primary concern is the choice of evaluation criteria. Attention must be paid to the fact that the criteria must not be correlated with performance units. DEA models give a set of recommendations in order to improve the efficiency of non-efficient units, by either increasing outputs or decreasing outputs. However, in agricultural context, some recommendations given by classical DEA models may be unrealistic for the following reasons: smaller production units are preferred, but the classical DEA models do not take into account the unique character of particular units. Therefore other (modified) DEA models are proposed for applications in agriculture.

As groundwork, Weighted Slack-Based Measures of efficiency (WSBM) proposed by Tone [1] and Generic Directional Distance Model (GDDM) proposed by Chambers [2] and [3] are used for evaluation of enterprises. An advantage of these modified DEA methods is the possibility of affecting recommended changes by weights or directional vectors. Further, WSBM and GDDM can be improved by the inclusion of negative data [4], [6].

The production conditions and utilization of agricultural land in the Czech Republic are characterized by farming areas and sub-areas depending on the climate and soil conditions. There are five production areas (maize growing region, sugar beet growing region, grain growing region, potato growing region and forage growing region), and each of them is further split into sub areas (21 total).

This article is focused on the potato growing region because that area constitutes the largest part of arable land in the Czech Republic. The area is characterized by typical altitudes of 400-650 m above the sea level, its total cultivated land is 60-80% and the average slope is 5-12%. Another reason for this choice was the homogeneity from the point of view of grain production structure.

MATERIAL AND METHOD

The above mentioned Weighted Slack-Based Measure of efficiency (WSBM) and Generic directional distance model (GDDM) seemed very suitable.

Weighted Slack-Based Measure of Efficiency (WSBM) model

The WSBM can be applied to any situation with a priori defined weights of inputs and outputs. This model was derived from the SBM model, see Tone [5] and was further improved see Tone and Tsutsui [6] in order to permit negative data in the dataset.

In case VRS the WSBM model considering weights of inputs and outputs is introduced as follows:

$$\min\left\{\frac{1-\frac{1}{m}\sum_{i=1}^{m} w_{i0} s_i^- / x_{i0}}{1+\frac{1}{s}\sum_{k=1}^{s} r_{k0} s_k^+ / y_{k0}} \middle| \mathbf{x_0} = \mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^-,\ \mathbf{y_0} = \mathbf{Y}\boldsymbol{\lambda} - \mathbf{s}^+,\ \boldsymbol{\lambda} \ge \mathbf{o}, \mathbf{s}^- \ge \mathbf{o}, \mathbf{s}^+ \ge \mathbf{o},\ \sum_{j=1}^{n} \lambda_j = 1\right\}, \quad (1)$$

where w_{i0}, r_{k0} are weight for input *i* and output *k* of DMU_0. For a detailed description see Tone [5] and [1].

Generic Directional Distance Model (GDDM)

The other possibility of influencing changes in the size of inputs and outputs is represented with the Generic Directional Distance Model (GDDM).

This model was proposed by Chambers [2] and [3]. The main advantage of this model is the ability to project inefficient units on the efficiency frontier with a selected direction. This feature is particularly attractive applicable in our situation, because the model provides each unit with its own way to efficiency, respecting its improvement potential. The GDDM is as follows:

For the set of unit $j = 1,2,...,n$, with input levels $x_{ij}, i = 1,2,...,m$ and output levels $y_{kj}, k = 1,2,...,s$ and unit $o \in j$ which is to be assessed is the generic directional distance model given as follows:

$$\max \left\{ \begin{array}{l} \beta_o \mid \sum_{j=1}^{n} \lambda_j y_{kj} \geq y_{ko} + \beta_o g_{y_k}, k = 1,2,...,s, \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta_o g_{x_i}, i = 1,2,...,m, \sum_{j=1}^{n} \lambda_j = 1, \\ \lambda_j, \beta_o, g_{x_i}, g_{y_k} \geq 0 \end{array} \right\}, \quad (2)$$

where the vector $\mathbf{g}_x$ $(\mathbf{g}_y)$ represents possible changes of input (output).

The model (2) is valid in the case of variable returns to scale (VRS). Target values of inputs (outputs) were obtained as the product of **X (Y)** and **λ**. If negative data occurs among observed inputs (outputs), the last constraint of model (2) would be violated.

GDDM was further modified by Portela et al. [4] in order to handle negative data for the Range Directional Model (RDM+) and Inverse Range Directional Model (RDM-). These models substitute the vector of desired changes with the vector of inputs and output ranges in the case of RDM+ and the vector of the inverted value of the ranges in the case of RDM-. Both types of RDM use different ways to compute efficiency scores similarly to radial efficiencies traditionally used in DEA (see Portela et al. [4]).

Efficiency scores for the standardized model can be calculated as follows.

$$\varphi_o = 1 - \frac{\sqrt{\sum_{i=1}^{m} \left(x_{io}^* - x_{io}\right)^2 + \sum_{k=1}^{s} \left(y_{ko}^* - y_{ko}\right)^2}}{\sqrt{\sum_{i=1}^{m} R_{io}^{\ 2} + \sum_{k=1}^{s} R_{ko}^{\ 2}}}, \quad (3)$$

where

φ_o is efficiency for assessed unit *o*;

x_{io}^* is the target value of *i*-th input projected on the efficiency frontier;

y_{ko}^* is the target value of *k*-th output projected on the efficiency frontier.

$$R_{ko} = \max_j \{y_{kj}\} - y_{ko}, k = 1,2,...,s,$$

$$R_{io} = x_{io} - \min_j \{x_{ij}\}, i = 1,2,...,m.$$

The main advantage of this model is the ability to project inefficient units on the efficiency frontier with the selected direction and to obtain a most realistic recommendation. This feature is well applicable in our application, because the model provides each unit with its own way to efficiency that respects its improvement potential.

RESULTS AND DISCUSSIONS

We considered a group of 25 farms of similar characteristics (in the potato growing region). As inputs, we took the ratio of land area designated for growing wheat, barley and the rape. As outputs, we took yield of the afore-mentioned crops in tons per hectar (see *table 1*).

Table 1

Description of inputs and outputs

Symbol	Description
x_1	ratio of soil for growing wheat
x_2	ratio of soil for barley
x_3	ratio of soil for rape
y_1	yield per hectare of growing wheat [t]
y_2	yield per hectare barley [t]
y_3	yield per hectare rape [t]

Input data are presented in *table 2*.

Table 2

Inputs and outputs

	Inputs			Outputs		
j	x_1	x_2	x_3	y_1	y_2	y_3
1	0.264	0.106	0.191	6.30	5.30	3.50
2	0.235	0.237	0.150	5.29	4.11	2.49
3	0.461	0.259	0.278	5.39	3.82	3.04
4	0.300	0.216	0.138	3.89	3.81	2.76
5	0.257	0.225	0.067	7.07	6.74	3.76
6	0.323	0.141	0.039	7.80	6.71	3.60
7	0.330	0.128	0.111	6.39	5.15	3.26
8	0.392	0.171	0.163	5.54	3.82	3.82
9	0.353	0.231	0.076	5.33	4.65	2.89
10	0.270	0.239	0.126	6.92	5.52	3.43
11	0.326	0.020	0.150	5.65	5.77	3.44
12	0.307	0.156	0.086	7.56	6.29	4.21
13	0.341	0.377	0.000	5.87	4.49	0.00
14	0.338	0.307	0.131	5.71	4.13	3.04
15	0.315	0.241	0.082	5.25	4.65	3.49
16	0.445	0.000	0.131	4.94	0.00	4.54
17	0.254	0.045	0.179	4.40	2.98	2.36
18	0.370	0.075	0.063	5.52	5.53	3.46
19	0.318	0.091	0.140	6.53	5.27	3.70
20	0.288	0.216	0.072	6.33	5.04	3.82
21	0.379	0.098	0.183	5.79	5.63	3.64
22	0.343	0.077	0.113	5.33	4.55	3.20
23	0.208	0.160	0.143	5.52	4.79	3.65
24	0.144	0.122	0.058	3.91	5.00	3.71
25	0.197	0.188	0.155	4.05	4.35	2.53

Determination of weights for evaluation models

The first step to successful use of the above-mentioned models is to determine the direction of the potential improvement described by vectors of weights for the WSBM and the directional vector for the GDDM.

For the outputs represented by per-hectare yields, we adjust the weights according to the price level of crop plants produced by particular farmer. Values of per hectare profits are proposed for the input weights. But weights for inputs are established to the contrary to weights for outputs. Consequently for the outputs the biggest emphasis is placed on the crop with the highest price. For the inputs the biggest emphasis is placed on the reduction of the crop with the lowest per hectare profit. In the special case of negative profit of two particular crops and one positive profit, the vector of potential improvement was set to (0.5; 0.5; 0). This situation occurred for unit 3 and many other units, see *Table 3*. Likewise this adjustment ensures binding of inputs and outputs which are typical in agriculture.

Table 3

Determination of weights for evaluation model

	Inputs			Outputs		
j	x_1	x_2	x_3	y_1	y_2	y_3
1	0.367	0.372	0.260	0.26	0.22	0.53
2	0.478	0.457	0.065	0.23	0.24	0.53
3	0.500	0.500	0.000	0.21	0.21	0.58
4	0.500	0.500	0.000	0.21	0.25	0.54
5	0.365	0.500	0.135	0.26	0.25	0.49
6	0.442	0.417	0.141	0.24	0.25	0.51
7	0.490	0.500	0.010	0.23	0.24	0.53
8	0.500	0.500	0.000	0.21	0.22	0.57
9	0.014	0.500	0.486	0.25	0.22	0.53
10	0.500	0.500	0.000	0.23	0.25	0.52
11	0.194	0.306	0.500	0.26	0.21	0.53
12	0.448	0.052	0.500	0.22	0.32	0.47
13	0.000	0.500	0.500	0.52	0.48	0.00
14	0.407	0.358	0.235	0.24	0.25	0.51
15	0.248	0.500	0.252	0.22	0.24	0.54
16	0.500	0.500	0.000	0.25	0.00	0.75
17	0.428	0.275	0.297	0.21	0.29	0.50
18	0.471	0.500	0.029	0.22	0.20	0.58
19	0.286	0.379	0.335	0.26	0.23	0.51
20	0.332	0.500	0.168	0.25	0.24	0.51
21	0.120	0.500	0.380	0.22	0.25	0.54
22	0.382	0.463	0.155	0.24	0.20	0.56
23	0.500	0.000	0.500	0.22	0.26	0.52
24	0.324	0.453	0.223	0.24	0.22	0.54
25	0.405	0.500	0.095	0.25	0.21	0.54

Results for WSBM and GDDM are depicted in *tables 4* and *5*. There is clear that the particular model gives different recommendation.

Table 4

Results for WSBM model for inefficient units

	Inputs			Outputs		
j	x_1	x_2	x_3	y_1	y_2	y_3
2	0.195	0.165	0.078	5.29	6.76	5.02
3	0.311	0.019	0.143	5.39	5.50	3.28
4	0.143	0.121	0.058	3.89	4.97	3.69
7	0.324	0.062	0.111	6.39	6.09	3.49
8	0.355	0.026	0.163	6.20	6.37	3.82
9	0.259	0.113	0.031	6.26	5.39	2.89
10	0.255	0.216	0.103	6.92	8.85	6.57
12	0.307	0.156	0.052	7.56	7.12	4.21
14	0.210	0.178	0.085	5.71	7.30	5.42
15	0.313	0.137	0.038	7.56	6.50	3.49
17	0.254	0.016	0.117	4.40	4.49	2.68
19	0.318	0.091	0.123	6.53	6.88	4.35
20	0.243	0.168	0.072	6.33	7.16	4.92
21	0.379	0.020	0.164	6.19	5.63	3.98
22	0.302	0.036	0.113	5.33	4.74	3.20
23	0.208	0.160	0.073	5.52	6.67	4.78
25	0.149	0.126	0.060	4.05	5.18	3.84

Table 5

Results for GDDM model for inefficient units

	Inputs			Outputs		
j	x_1	x_2	x_3	y_1	y_2	y_3
2	0.193	0.169	0.050	5.31	5.06	2.82
3	0.266	0.064	0.153	5.47	5.14	3.26
4	0.167	0.083	0.090	3.94	4.19	2.91
7	0.293	0.090	0.110	6.41	5.74	3.36
8	0.287	0.066	0.130	5.58	6.10	3.94
9	0.264	0.115	0.032	6.37	5.48	2.94
10	0.252	0.220	0.066	6.93	6.61	3.68
12	0.306	0.156	0.085	7.56	7.00	4.21
14	0.210	0.184	0.056	5.79	5.61	3.20
15	0.277	0.135	0.043	6.78	6.21	3.57
17	0.234	0.032	0.122	4.41	4.28	2.62
19	0.312	0.083	0.133	6.54	6.14	3.71
20	0.251	0.150	0.053	6.36	6.31	3.88
21	0.363	0.034	0.135	5.89	5.66	3.71
22	0.315	0.044	0.102	5.35	4.56	3.28
23	0.207	0.160	0.082	5.52	5.69	3.65
25	0.149	0.129	0.047	4.08	4.37	2.79

A comparison for two specific DMUs can be found in *Table 6* and *7*. Somewhat surprisingly neither model respects the given improvement vector in the obtained recommendations. This situation can be explained the following way:

only eight units are efficient and many units are not and thus a significant part of units is not covered by efficiency envelopment. For those units the requested reduction of inputs and extension of outputs can not be fulfilled.

The efficiency scores for the both compared units are shown in *tables 6* and *7*. For the GDDM was computed score by the formula (3). The ranking of the units according their efficiency is more describing than the efficiency score. Rank of DMUs is not very different for compared units 7 and 15 as well as for the rest of units.

The changes proposed by WSBM for DMU 7 are obviously more considerable than the changes proposed by GDDM. In general we can say that the recommendation of the GDDM is more moderate than the recommendation of the WSBM, the average percentage of the change is 25.8 % for WSBM (28.7% – input changes, 22.9 % output changes) and 19.4 % for GDDM (29.1% – input changes, 9.8 % output changes). For these reasons, GDDM can be recommended if more moderate or balanced changes are needed.

Table 6

Comparison relative differences between models for unit 7

	x_1	x_2	x_3	y_1	y_2	y_3	eficiency	rank
Weights / vector components	0.49	0.50	0.01	0.23	0.24	0.53	-	-
WSBM	2%	51%	0%	0%	18%	7%	0.680	12
GDDM	11%	30%	1%	0%	12%	3%	0.758	14

Table 7

Comparison relative differences between models for unit 15

	x_1	x_2	x_3	y_1	y_2	y_3	eficiency	rank
Weights / vector components	0.248	0.500	0.252	0.22	0.24	0.54	-	-
WSBM	1%	43%	54%	44%	40%	0%	0.542	17
GDDM	12%	44%	48%	29%	34%	2%	0.370	24

CONCLUSIONS

In this paper we tested two additive DEA models with the aim to find realistic recommendations for agricultural enterprises where classical DEA models fail to respect the particular improvement which takes into account existing results of farming and market conditions. As this projection on the efficiency frontier is generally non-radial, we propose a new method to compute an efficient score too.

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