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ABSTRACTS

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CONTENTS

I. Abstracts of Invited Lectures

Group actions on generalized trees: median and metric viewpoints

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The aim of the talk is to introduce the main concepts and state some results and directions of research concerning actions on trees and their generalizations (median sets (algebras), Λ-trees, where Λ is an abelian lattice ordered group, in particular, a totally ordered abelian group). Some combinatorial as well some more algebraic-geometric techniques (as the duality theory for median sets) will be combined to sketch the proof of an embedding theorem for free actions on median sets.

Univalence conditions for integral operators with applications in fluid mechanics

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Univalence conditions have been the subject of research of many romanian and international mathematics researchers. The integral operators have been introduced and studied already at the beginning of 1915 by Alexander. Since then these operators have been extended by many mathematicians. Also various properties and univalence criteria were obtained. Sufficient univalence conditions proved their aplicability in fluid mechanics, more precisely in solving boundary inverse problems.

On solutions of perturbed optimization problems

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In the present article we present conditions under which the set of continuous perturbations of a given lower semi-continuous function attains minimum on a subset with concrete properties is "big" in a topological sense.

By a space we understand a completely regular topological Hausdorff space. We use the terminology from [2, 5]. Let $\mathbb{R}$ be the space of reals and $\mathbb{R}^\infty = \mathbb{R} \cup \{+\infty\}$.

Let X be a topological space.

Denote by $C(X)$ the Banach space of all bounded continuous functions $f : X \longrightarrow \mathbb{R}$ with the sup-norm $\| f \| = sup\{|f(x)|; x \in X\}$.

For any function $\psi : X \longrightarrow \mathbb{R}^\infty$ and $Y \subseteq X$ we put $inf_Y(\psi) = inf\{\psi(x) : x \in Y\}$, $m_Y(\psi) = \{x \in Y : \psi(x) = inf_Y(\psi)\}$ and $dom(\psi) = \{x \in X : \psi(x) < +\infty\}$.

The function ψ is called *proper* if hers domain $dom(\psi)$ is non-empty.

A minimization problem (X, ψ) is called:

- Tychonoff well-posed if $m_X(\psi)$ is a singleton and every minimizing sequence $\{x_n \in X : n \in \mathbb{N}\}$ of the function ψ is convergent to a point from $m_X(\psi)$;

- almost-well-posed if every minimizing sequence $\{x_n \in X : n \in \mathbb{N}\}$ of the function ψ has a cluster point in X;

- weakly Tychonoff well-posed if $m_X(\psi)$ is a compact set and every minimizing sequence $\{x_n \in X : n \in \mathbb{N}\}$ of the function ψ has an accumulation point.

A function $f : X \longrightarrow \mathbb{R}^\infty$ is called *lower semi-continuous* (respectively, *upper semi-continuous*) if and only if the set $\{x \in X : f(x) > t\}$ (respectively, $\{x \in X : f(x) < t\}$) is an open set for every $t \in \mathbb{R}$.

Theorem 1. *Let $\psi : X \longrightarrow \mathbb{R}^\infty$ be a proper bounded from below lower semi-continuous function on a space X. Assume that in any non-empty closed in X subspace $Y \subseteq dom(f)$ some point is of countable pseudocharacter in Y. Then the set $\{f \in C(X) : m_X(f + \psi)$ is a singleton$\}$ is dense in E.*

Theorem 2. *Let $f : X \longrightarrow \mathbb{R}^\infty$ be a proper bounded from below lower semi-continuous function on a space X. Assume that in any non-empty closed in X subspace $Y \subseteq dom(f)$ some point is of countable character in Y. Then the set $\{f \in C(X) :$ the minimization problem $(X, f + \psi)$ is Tychonoff well-posed$\}$ is dense in $C(X)$.*

Theorem 3. *Let $f : X \longrightarrow \mathbb{R}^\infty$ be a proper bounded from below lower semi-continuous function on a space X and $dom(f)$ is a space with a complete base of countable order. Then the set $\{f \in C(X) :$ the minimization problem $(X, f + \psi)$ is Tychonoff well-posed$\}$ contains a G_δ-dense in $C(X)$ subset.*

Let B be a Banach space and $\Phi : B \longrightarrow C(X)$ be a continuous linear operator. For a proper bounded from below lower semi-continuous function $\psi : X \longrightarrow \mathbb{R}^\infty$ on a space X consider the following sets of continuous perturbations:

- $SM(\psi, B, \Phi) = \{b \in B : m_X(\Phi(b) + \psi)$ *is a singleton*$\}$;

- $TWP(\psi B, \Phi) = \{b \in B$: *the minimization problem* $(X, \Phi(b) + \psi)$ *is Tychonoff well-posed*$\}$;

- $aWP(\psi, B, \Phi) = \{b \in B$: *the minimization problem* $(X, \Phi(b) + \psi)$ *is almost-well-posed*$\}$;

- $wTWP(\psi, B, \Phi) = \{b \in B$: *the minimization problem* $(X, \Phi(b) + \psi)$ *is weakly Tychonoff well-posed*$\}$.

We present conditions under which the set of continuous perturbations of a given lower semi-continuous function attains minimum on a subset with concrete properties is "big" in a topological sense. These problems are typical for the distinct variational principles in optimization. Some optimization problems in topological spaces were studied in [1, 3, 4, 5].

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Continuum models in elasto-plastic materials with micro-structure

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We develop a mathematical framework able to describe the macroscopic behaviour of elasto-plastic materials, taking into account the presence of the defects existing at the micro structure level. Two type of forces, macro and micro, which obey their own balance equations are characterized in terms of stresses and momenta. The energetic arguments, as the principle of the free energy imbalance, provide thermomechanics restrictions on the constitutive framework. Material behaves like an hyperelastic (second order) in terms of macro forces. The micro stresses and micro stress momenta are related with the plastic type behaviour as well as with the microstructural defects, and they are represented under the viscoplastic type constitutive equations, which have to be compatible with the imbalance of the free energy. Within the constitutive framework of second order finite elasto-plasticity, for any motion and at every material particle an anholonomic (time dependent) configuration, $\mathcal{K}$, has been associated through a pair of plastic distortion $\mathbf{F}^p$ and plastic connection Γ^p, following Cleja-Tigoiu (2007) and (2010). Lattice defects are treated as differential geometrical aspects, in terms of the elements which enter the representation of the plastic connection. The linear approaches to the lattice defects, which are described by de Wit (1981), have a counterpart in finite elasto-plasticity. The dislocations can be represented by non-zero plastic torsion associated with Bilby connection, disclinations are modeled in our representation by a second order tensor Λ, which generates non-zero curvature. A quasi-dislocation density $\alpha^{\mathbf{Q}}$ is associated with the non-metricity of the plastic connection. The imbalance free energy principle is defined following Gurtin (2002) and Cleja-Tigoiu (2007), which is defined in terms of the free energy and internal power, expanded to include the work done by the forces conjugated with the appropriate rate of the elastic and plastic second order deformations, as well as with the rates of variables and thier gradients, respectively, which describe the presence of the appropriate defects. Finally the constitutive equations and flow rule-like conditions, compatible with the imbalanced free energy can be derived. A comparative study of various approaches to finite elasto - plasticity with continuously distributed defects is also presented.

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Graded subalgebras: Algebra, Topology, Geometry. II. Dedicated to the memory of Professor Nicolae RADU (1931-2001))

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This talk is mainly destined to the topological conditions of finite generation of $\mathbb{N}$-graded $\mathbb{C}$-subalgebras and it is a continuation of [2]. So we will consider here the case of the base field $k = \mathbb{C}$ (the complex number field) and of a $\mathbb{N}$-graded subalgebra $A = \oplus_{i=0}^{\infty} A_i$ of a $\mathbb{C}$-algebra of finite type (called shortly *a graded* $\mathbb{C}$*-subalgebra*). Then A_0 is a (ungraded) subalgebra of A and so a subalgebra of a finitely generated $\mathbb{C}$-algebra. According to *A. Grothendieck* ([5]), we may associate to A_0 and A the corresponding geometric objects: the affine $\mathbb{C}$-scheme $Spec\, A_0$ and the projective $\mathbb{C}$-scheme $Proj\, A$. We have a canonical $\mathbb{C}$-scheme morphism $\pi : Proj\, A \to Spec\, A_0$. Since $k = \mathbb{C}$, according to [3] we can consider on the subsets of all closed points of the schemes above, $(Spec\, A_0)_{cl} \subseteq Spec\, A$ and $(Proj\, A)_{cl} \subseteq Proj\, A$, the *Gel'fand* topologies, which are separated and finer that the supported *Zariski* topologies. We have an induced map $(\pi)_{cl} : (Proj\, A)_{cl} \to (Spec\, A_0)_{cl}$ which is continuous with respect to the *Gel'fand* topologies. The main fact is the following:

Let $A = \oplus_{i=0}^{\infty} A_i$ be a reduced $\mathbb{N}$-graded subalgebra of an algebra of finite type over $\mathbb{C}$. Then A is finitely generated over $\mathbb{C}$ if and only if the following conditions are fulfilled:

i) the Gel'fand topology on $(Spec\, A_0)_{cl}$ is locally compact,

ii) the canonical map $\pi_{cl} : (Proj\, A)_{cl} \to (Spec\, A_0)_{cl}$ is proper with respect to the Gel'fand topologies (in the meaning of [1], Ch.1, §10, Def.1), *iii) the scheme Proj A is quasi-compact* (with respect to the *Zariski* topology).

If $dim\, A \leq 2$, the last condition *iii) can be removed.*

In particular,

If $A_0 = \mathbb{C}$, *then A is finitely generated over* $\mathbb{C}$ *if and only if* $(Proj\, A)_{cl}$ *is compact with respect to the Gel'fand topology*

The previous equivalent conditions for the finite generation of A are obviously of *pure topological* nature.

Some consequences of the above author's results could be presented, including a natural and simple proof of a very known result of *Zariski* or a direct application to so-called *"Zariski and Constantinescu"* Theorem (MR1836861 (2002e:14022)), all of them for the situation when the base field $k = \mathbb{C}$.

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Espaces de Sobolev à exposants variables: aspects géométriques

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Soit $\Omega \subset R^N$ un domaine borné et régulier et $p \in C(\bar{\Omega}, R)$ satisfaisant $p(x) > 1$ pour tout $x \in \bar{\Omega}$. On montre que l'espace de Sobolev généralisé $(W_0^{1,p(\bullet)}, \|\ \|_{1,p(\bullet)})$ est lisse c'est à dire la norme $\|\|_{1,p(\bullet)}$ est Fréchet différentiable. Plus encore, si $p(x) \geq 2$ pour tout $x \in \bar{\Omega}$, alors $(W_0^{1,p(\bullet)}, \|\ \|_{1,p(\bullet)})$ est uniformément convexe. Des conséquences de ces propriétés sont mises en évidence.

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Accurate Spectral Solutions to some Non-Standard Eigenvalue Problems

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It is a matter of evidence that linear stability problems encountered in the mechanics of continua become more complex due to incorporation of technologically important effects. Consequently, the order of differential equations increases rapidly and the type of the boundary conditions attached diversifies. This happens in the theory of elasticity (see for example our monograph [1]) as well as in the thermo and hydrodynamics of viscous fluids (see for instance our recent papers [2] and [3]). Generally, the discretization methods applied to such problems lead to matrices which become less conditioned as both, the order of approximation, and the order of differentiation increase. This fact is particularly hindering for spectral collocation methods in spite of their high accuracy. Also, to enforce non homogeneous arbitrary boundary conditions it is a difficult task. In this context, the aim of this talk is two-fold. First, we introduce a method to solve high and even order two-point eigenvalue (boundary value) problems with Dirichlet and hinged (Navier) boundary conditions. It reduces the problem to a system of second order differential equations supplied only with Dirichlet boundary conditions. Further, accurate spectral methods are used (mainly of collocation type, see [4]) in order to solve such systems. Second, in order to prove its feasibility the method is used to solve various challenging problems such as hydrodynamic stability problems of Benard type, Viola's eigenvalue problem (a singularly perturbed one, see [5]), Mathieu eigenvalue problem as a two parameter eigenvalue problem, etc. A particular attention is paid to solution of the induced generalized algebraic eigenvalue problems. The second matrices in these problems are singular and consequently the classical QZ algorithm typically generate spurious eigenvalues. More than that, it takes considerable CPU time for large discretizations. To avoid such inconveniences we use subspace type methods (Jacobi-Davidson) in order to effectively solve these problems.

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A generalization of the dynamical model Euler-Lagrange and the problem of financing cosmic research

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Multi-objective decision making based on fuzzy events and their coherent fuzzy measures

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In this paper, the multiobjective decision making problem is formulated in terms of fuzzy scores, their coherent defuzzification (from the logical point of view taken into account), and aggregation of defuzzified values. The objectives are seen as subsets (or events) of a universal set U and the extent to which an alternative Ai satisfies ij an objective Oj is a fuzzy event conditional on Oj, i.e., a fuzzy set ij are ij of Oj. The elements of the partition defined on a finite partition ij in an element x particular aspects of the objective Oj, the value assumed by ij is the extent to which Ai satisfies that particular aspect. Using a of procedure of defuzzification, fuzzy scores of alternatives with respect to an objective are transformed into numerical scores. Defuzzified scores are also bound by the conditions of consistency, they must take account of the logical relationships between the various events of the partitions of Oj obtained by varying Ai. In the paper we study the conditions of consistency of defuzzified scores. Finally, for each alternative Ai are developed criteria for the aggregation of defuzzified scores, taking into account the logical relations between the objectives. Applications to Social Sciences are sketched.

Uniformization of the sphere and flat singular surfaces

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We construct flat metrics in a given conformal class with prescribed singularities of real orders at marked points of a closed real surface. The singularities can be small conical, cylindrical, and large conical with possible translation component. Along these lines we give an elementary proof of the uniformization theorem for the sphere.

Reserved subject

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Model Order Reduction: mathematical methods and applications

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Model Order Reduction (MOR) is a flourishing field in numerical mathematics that aims at reducing complex models while retaining dominant features, as well as relevant properties. It originates from the systems and control discipline, the most popular technique being truncated balanced realization that is based on the solution of systems of Lyapunov equations. Since the 1990's, however, numerical mathematicians became interested in the field, especially after the breakthrough work of Feldmann and Freund on using Lanczos methods to generate low order models. This has led to a wealth of developments, to date still mainly for linear models, but also for the nonlinear and parameterized case. There is an intimate relation with numerical linear algebra, most notably the solution of large linear systems and the determination of selected eigenvalues. In this presentation, we will discuss the most important developments in Model Order Reduction to date from a numerical point of view. Lanczos and Arnoldi type methods, the dominant and sensitive pole algorithms, efficient solution of large Lyapunov systems will be touched upon. In addition, a number of applications in industry will be shown, MOR being of vital importance for challenging simulations.

II. Abstracts of Short Communications

Section 1
Dynamical systems, jets theory and their applications; quadratic dynamical systems

Classification of homogeneous quadratic differential systems on $\mathbb{R}^3$ having a derivation

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A commutative binary algebra is associated with each such a homogeneous quadratic differential system (HQDS). There exists a 1-1 correspondence between the family of all classes of affinely equivalent homogeneous quadratic differential systems and the set of all classes of isomorphic commutative algebras defined on the same space as the associated systems. Consequently the problem of classifying HQDSs up to an affinity becomes equivalent to problem of classifying, up to an isomorphism, of commutative algebras. Notice that any derivation (resp. automorphism) of a HQDS is a derivation (resp. automorphism) of its associated algebra. In our paper we shall classify, up to an affinity, the HQDSs defined on $\mathbb{R}^3$ and having a derivation. It was proved:

(i) there exists 20 classes of (non-null) algebras which are nonisomorphic each other; Consequently, there exist 20 classes of homogeneous quadratic (nontrivial) differential systems affinely nonequivalent each other.

(ii) there exist fifty-three classes of mutually nonisomorphic commutative algebras having a nilpotent of order two derivation.

(iii) there exist nine classes of mutually nonisomorphic commutative algebras having a nilpotent of order three derivation.

(iv) there exist eighteen classes of mutually nonisomorphic commutative algebras having a semisimple derivation with a 2-dimensional kernel.

(v) there exist twenty-one classes of mutually nonisomorphic commutative algebras having a semisimple derivation with a 2-dimensional kernel.

(vi) there exist three classes of mutually nonisomorphic commutative algebras having a semisimple nonsingular derivation.

A study of commutative algebras without derivations is manly based on them associated (algebraic) homogeneous systems.

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The transvectants and $GL(2,\mathbb{R})$-invariant center conditions for some cubic differential systems

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Let us consider the cubic system of differential equations

$$\frac{dx}{dt} = P_1(x,y) + P_2(x,y) + P_3(x,y), \quad \frac{dy}{dt} = Q_1(x,y) + Q_2(x,y) + Q_3(x,y), \quad (1)$$

where $P_i(x,\ y)$, $Q_i(x,\ y)$ are homogeneous polynomials of degree i in x and y with real coefficients. The $GL(2,\mathbb{R})$-comitants [1] of the first degree with respect to the coefficients of system (1) have the form

$$R_i = P_i(x,y)y - Q_i(x,y)x,\ S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right),\ i = 1,2,3. \quad (2)$$

From the classical invariant theory [2] the definition of the transvectant of two polynomials is well known.

Definition 1. *Let $f(x,y)$ and $\varphi(x,y)$ be homogeneous polynomials in x and y with real coefficients of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, and $k \in \mathbb{N}^*$. The polynomial*

$$(f,\varphi)^{(k)} = \frac{(\rho-k)!(\theta-k)!}{\rho!\theta!}\sum_{h=0}^{k}(-1)^h\binom{k}{h}\frac{\partial^k f}{\partial x^{k-h}\partial y^h}\frac{\partial^k \varphi}{\partial x^h \partial y^{k-h}}$$

is called the transvectant of the index k of polynomials f and φ.

Remark 1. *If the polynomials f and φ are $GL(2,\mathbb{R})$-comitants of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, for the system (1), then the transvectant of the index $k \leq \min(\rho,\theta)$ is a $GL(2,\mathbb{R})$-comitant of the degree $\rho+\theta-2k$ for the system (1). If $k > \min(\rho,\theta)$, then $(f,\varphi)^{(k)} = 0$.*

In repeated using of the transvectants a set of the parenthesis of the type $((\ldots($ will be replaced by a single parenthesis of the form $[\![$. By using the transvectants for the system (1) the following $GL(2,\mathbb{R})$-invariants were constructed:

$$H_1 = 3(R_2,R_1)^{(2)}, \quad K_1 = 3(R_2,S_3)^{(2)}, \quad I_1 = S_1, \quad I_2 = (R_1,R_1)^{(2)},$$
$$I_3 = [\![R_2,R_1)^{(2)},S_2)^{(1)}, \quad I_4 = [\![R_1,S_2)^{(1)},S_2)^{(1)}, \quad I_5 = [\![R_2,R_2)^{(2)},R_1)^{(2)},$$
$$I_6 = [\![R_2,R_1)^{(2)},R_1)^{(1)},S_2)^{(1)}, \quad I_7 = [\![R_2,R_2)^{(2)},S_2)^{(1)},S_2)^{(1)},$$
$$I_8 = [\![R_2,R_2)^{(2)},R_2)^{(1)},R_2)^{(3)}, \quad I_9 = [\![R_2,S_2)^{(1)},S_2)^{(1)},S_2)^{(1)},$$
$$I_{10} = [\![R_2,R_1)^{(2)},R_1)^{(1)},H_1)^{(1)}, \quad I_{11} = [\![R_2,R_2)^{(2)},R_2)^{(1)},R_1)^{(2)},S_2)^{(1)},$$
$$I_{12} = [\![R_2,R_2)^{(2)},R_1)^{(1)},S_2)^{(1)},S_2)^{(1)}, \quad I_{13} = [\![R_2,R_1)^{(1)},S_2)^{(1)},S_2)^{(1)},S_2)^{(1)},$$
$$I_{14} = [\![R_2,R_2)^{(2)},R_2)^{(1)},R_1)^{(2)},R_1)^{(1)},S_2)^{(1)},$$
$$J_1 = (R_1,S_3)^{(2)}, \quad J_2 = (S_3,S_3)^{(2)},$$
$$J_3 = [\![R_2,R_2)^{(2)},S_3)^{(2)}, \quad J_4 = [\![R_2,S_3)^{(2)},S_2)^{(1)}, \quad J_5 = [\![S_3,S_2)^{(1)},S_2)^{(1)},$$
$$J_6 = [\![R_2,R_2)^{(2)},R_1)^{(1)},S_3)^{(2)}, \quad J_7 = [\![R_2,R_1)^{(2)},S_3)^{(1)},S_2)^{(1)},$$

$$J_8 = [\![R_2, S_3)^{(2)}, R_1)^{(1)}, S_2)^{(1)}, \quad J_9 = [\![R_1, S_3)^{(1)}, S_2)^{(1)}, S_2)^{(1)},$$
$$J_{10} = [\![R_2, S_3)^{(2)}, S_3)^{(1)}, S_2)^{(1)}, \quad J_{11} = [\![R_2, R_1)^{(2)}, R_1)^{(1)}, K_1)^{(1)},$$
$$J_{14} = [\![R_2, R_1)^{(2)}, R_1)^{(1)}, S_3)^{(1)}, S_2)^{(1)},$$
$$J_{19} = [\![R_2, R_1)^{(2)}, R_1)^{(1)}, S_3)^{(1)}, H_1)^{(1)},$$
$$J_{20} = [\![R_2, R_1)^{(1)}, S_3)^{(2)}, S_3)^{(1)}, H_1)^{(1)},$$
$$L_1 = 12I_{12} + 8I_{13} + 3(12I_3 + I_4)J_1 - 2I_2(9J_3 + 6J_4 - 2J_5) + 36J_{14},$$
$$L_2 = (165I_3 - 914I_4 - 1755I_5)I_{10} + 6I_2^2(93I_7 - 270I_8 + 4I_9 + 81J_6 + 288J_7 +$$
$$126J_8 - 135J_9) - 3I_2(489I_3^2 - 40I_3I_4 + 507I_3I_5 + 268I_4I_5 - 1458I_{14} + 243J_{19}),$$
$$L_3 = 2I_2(48I_3 - 16I_4 - 9I_5) + 45I_{10}, \quad G_1 = 4I_6 + 3I_2J_1,$$
$$G_2 = 4L_1 - 180I_{11} + 120I_{12} - 45(I_3 - 6I_5)J_1 + 5I_2(9J_3 + 24J_4 + 4J_5) + 90J_{11},$$
$$G_3 = -2L_1(2I_2I_3 + 9I_2I_5 - 7I_{10}) - 18I_2^2(58I_3 - 61I_4 - 24I_5)J_3 -$$
$$6I_2^2(23I_3 - 60I_4 - 48I_5)J_4 + 4I_2^2(43I_3 + 16I_4 + 48I_5)J_5 -$$
$$3I_2^2J_1(186I_7 + 88I_9 - 378J_7 + 207J_8 + 6J_9) - 27I_2^2(3I_2J_1J_2 - 8I_2J_{10} + 8J_{20}),$$
$$G_4 = L_1(16(105I_3 - 286I_4 - 945I_5)I_{10} +$$
$$9I_2^2(-16I_7 - 945I_8 - 64I_9 + 27I_2J_2 + 396J_6 + 1296J_7 + 720J_8 - 624J_9) -$$
$$3I_2(1776I_3^2 - 512I_3I_4 + 8268I_3I_5 - 1696I_4I_5 - 4725I_5^2 - 6552I_{14} + 1602J_{19})),$$
$$G_5 = L_1L_2L_3.$$

By using the comitants R_i and S_i $(i = 1, 2, 3)$ the system (1) can be written in the form

$$\frac{dx}{dt} = \frac{1}{2}\frac{\partial R_1}{\partial y} + \frac{1}{2}S_1x + \frac{1}{3}\frac{\partial R_2}{\partial y} + \frac{2}{3}S_2x + \frac{1}{4}\frac{\partial R_3}{\partial y} + \frac{3}{4}S_3x,$$

$$\frac{dy}{dt} = -\frac{1}{2}\frac{\partial R_1}{\partial x} + \frac{1}{2}S_1y - \frac{1}{3}\frac{\partial R_2}{\partial x} + \frac{2}{3}S_2y - \frac{1}{4}\frac{\partial R_3}{\partial x} + \frac{3}{4}S_3y.$$

In [3] for the cubic systems (1) with $I_1 = 0$, $I_2 > 0$, $R_3 \equiv 0$ the center conditions for the point $(0, 0)$ were constructed in the terms of the coefficients of the normal form of the system. In this paper the $GL(2, \mathbb{R})$-invariant center conditions for the point $(0, 0)$ for the cubic differential systems (1) with $I_1 = 0$, $I_2 > 0$, $R_3 \equiv 0$ were constructed.

Theorem 1. *The system* (1) *with the conditions* $I_1 = 0$, $I_2 > 0$, $R_3 \equiv 0$ *has the center in the origin of the coordinates if and only if the following conditions are fulfilled:*

$$G_1 = G_2 = G_3 = G_4 = G_5 = 0.$$

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Center and reversibility in a cubic differential system with one invariant straight line

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We consider the real cubic system of differential equations

$$\dot{x} = -y + P_2(x,y) + P_3(x,y), \quad \dot{y} = x + Q_2(x,y) + Q_3(x,y), \tag{1}$$

where $P_j(x,y)$, $Q_j(x,y)$ are homogeneous polynomials of degree j. The origin $O(0,0)$ is a singular point with purely imaginary eigenvalues, i.e. a weak focus. The problem of the center was completely solved for cubic systems with at least three invariant straight lines (see, for example, [1], [2]). In this paper we solve the problem of the center for cubic system (1) with one real invariant straight line by using the method of rational reversibility. First we find coefficient conditions for the existence of one real invariant straight line parallel to the axes Oy, i.e.

$$k = -a, \quad m = -c-1, \quad p = -f, \quad r = 0. \tag{2}$$

Next assuming that (2) holds we consider the rational transformation of the form

$$x = \frac{A_1X + B_1Y}{A_3X + B_3Y - 1}, \quad y = \frac{A_2X + B_2Y}{A_3X + B_3Y - 1} \tag{3}$$

with $A_1B_2 - B_1A_2 \neq 0$ and A_j, $B_j \in \mathbb{R}$, $j = 1,2,3$. The condition $A_1B_2 - B_1A_2 \neq 0$ guarantees that (3) is invertible in a neighborhood of $O(0,0)$ and the singular point $O(0,0)$ is mapped to $X = Y = 0$. We prove that A_j, B_j can be chosen so that the transformation (3) brings in some neighborhood of $O(0,0)$ the system (1) to one equivalent with a polynomial system

$$\frac{dX}{dt} = Y + M(X^2, Y), \quad \frac{dY}{dt} = -X(1 + N(X^2, Y)). \tag{4}$$

The obtained system has an axis of symmetry $X = 0$ and therefore $O(0,0)$ is a center for (1). So, for cubic system (1) with one real invariant straight line and a singular point a weak focus it was found coefficient conditions for (1) to be rationally reversible and $O(0,0)$ to be a center.

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A Hillam algorithm to locate fixed points

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Since locating all the fixed points of a discrete-time dynamical system involves the numerical solution of a simultaneous equations, we focused on root-finding algorithms. Hillam method will be presented for accomplishing this task. Along the way we will also see the manner in which Maple can be used to explore a complex problem.

On the computation of the term $w_{21}u^2\overline{u}$ of the series defining the center manifold for a scalar delay differential equation

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In computing the first several terms of the series of powers of the center manifold at an equilibrium point of a scalar delay differential equation of the type $\dot{x}(t) = f(x(t), x(t-r)),\ r > 0$, some problems occur at the term $w_{21}u^2\overline{u}$. More precisely, in order to determine the values at 0, respectively $-r$ of the function $w_{21}(\,.\,)$, an algebraic system of equations must be solved. We show that the two equations of this system are dependent, hence the system has an infinity of solutions. A method to overcome this lack of uniqueness is presented.

Dynamics of motion of hysteresis elastic characteristic plate

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This work dedicated that to learn dynamics of motion of hysteresis elastic characteristic plate which is protected from vibrations for harmonic and random processes. Vibroprotective object is liquid section and hysteresis type elastic dissipative characteristic dynamic absorber.

Probabilistic characterization of the dynamical stochastic systems with final critical state

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In this paper the dynamical stochastic systems with final critical state are studied. For these systems the set of states is finite and the transition time of the system from one state to another is considered unitary. At every discrete moment of time the distribution of the states is known. The evolution of the system is finishing when the system remain for m consecutive moments of time in his given critical state, where m is a fixed natural number. Such systems are a particular case of the dynamical systems with final sequence states. For these systems are defined some descriptive random variables: the evolution time of the system, the first alert moments and the total alert time. In this scientific research a new numerical approach for determining the main probabilistic characteristics of these descriptive random variables are proposed and grounded. The computational complexity of the elaborated algorithms is polynomial.

Regime switches induced by supply-demand equilibrium: a model for power prices dynamics

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Modelling electricity prices dynamics in deregulated power markets is a dicult task: power prices appear variable and unpredictable with jumps and spikes, and high, non constant, volatility. Empirical distributions of log-returns are characterized by large values of the standard deviation as well as non-zero skewness and very high kurtosis. We firstly discuss the use of reduced-form methodology to describe the dynamics of electricity prices in order to capture the statistical properties observed in real markets. Particular attention will be devoted to regime-switching models which seem good candidates to incorporate the main features of power prices as the seasonality component, the occurrence of stable and turbulent periods, as well as jumps and spikes. Regime-switching models offer, indeed, the possibility to introduce various mean-reversion rates and volatilities depending on the state of the system thus enhancing the exibility of the reduced-form approach. In the second part we show that regime switching dynamics of power prices can be obtained, in an equilibrium context, if we assume that the functional form of the supply curve is described by a power law in which the exponent is a two-state Markov process. This mechanism is responsible for random switches between regimes and it allows to describe the

main features of the price formation process. By the interplay between demand and supply, the proposed methodology can be used to capture shortages in electricity generation, forced outages, as well as peaks in electricity demand. An empirical analysis performed on market data is discussed in order to test the adaptability of the models in replicating the first four moment of the observed log-returns distributions.

The differential system (1,4) and algebraically independent focal pseudo-quantities

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We examine the system of differential equations $s(1,4)$ of the form

$$\dot{x} = cx + dy + gx^4 + 4hx^3y + 6ix^2y^2 + 4jxy^3 + ky^4,$$
$$\dot{y} = ex + fy + lx^4 + 4mx^3y + 6nx^2y^2 + 4oxy^3 + py^4 \qquad (1)$$

with Lyapunov's function [1]

$$U(x,y) = K_2 + \sum_{k=3}^{\infty} f_k(x,y),$$

where $K_2 = (cx+dy)y - (ex+fy)x$ is a center-affine comitant [2] of the system (1), and $f_k(x,y)$ are homogeneous polynomials of degree k with respect to x and y.

Assume that for the system (1) exist so constants $G_1, G_2, G_3, ...$ that the identity take place

$$\frac{dU}{dt} = \sum_{k=2}^{\infty} G_{k-1} K_2^k,$$

With the aid of the last equality we show that the constants $G_1, G_2, G_3, ...$ generates isobar polynomials [3] of the coefficients of the system (1) (called the focal pseudo-quantities of given system), which are the coefficients of some center-affine comitants (unimodulars) [2],[4] of given type [4], which belong to some finite dimensional linear spaces of comitants of the same type. Considering the space $V_{1,4}$ as a direct sum of these spaces then the generalized generating function of mentioned space will be write

$$\Phi(V_{1,4}, u, b, e) = 1 + b + 153u^4b^{14}e^2 + 4589u^6b^{42}e^4 + 49632u^8b^{85}e^6 + ...$$
$$+ C_{2(k+1),\frac{1}{2}(15k^2+11k+2),2k} u^{2(k+1)} b^{\frac{1}{2}(15k^2+11k+2)} e^{2k} + ...,$$

where

$$\left(2(k+1), \frac{1}{2}(15k^2+11k+2), 2k\right)$$

is the type of comitants in the respective space that corespond to the quantities G_{3k}, i.e. $2(k+1)$ is the degree of homogeneity of comitants with respect to x and y, $\frac{1}{2}(15k^2+11k+2)$ is the degree of homogeneity of comitants in relation to coefficients of linear part of the system (1) and $2k$ is the degree of homogeneity of comitants in relation to coefficients of cubic part of the system(1), $C_{2(k+1),\frac{1}{2}(15k^2+11k+2),2k}$ is unknown coefficient, $G_n = 0$ if $n \neq 3k$.

Using this function and generalized Hilbert series $H(S_{1,4}, u, b, e)$ from [4] the common Hilbert series $H_{S'_{1,4}}(t)$ of graded algebra $S'_{1,4}$ is built, which is generated by the space $V_{1,4}$. Studying the Krull dimension of algebra $S'_{1,4}$ with the aid of Hilbert series $H_{S'_{1,4}}(t)$ we obtain

Theorem. *The Krull dimension of algebra* $S'_{1,4}$ *not exceed* 13.

Corolary. *The maximal number of algebraically independent focal pseudo-quantities of system* $s(1,4)$ *from* (1) *not exceed* 13.

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Control of Hopf bifurcations in finite dimensional dynamical systems

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The paper presents a method to control Hopf bifurcations in a finite dimensional continuous dynamical system. Using an appropriate control low we show how such a control system exhibits controllable non degenerated or degenerated Hopf bifurcations.

Quadratic vector fields on $\mathbb{R}^3$ invariant under the D2 symmetry group

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Symmetry often plays an important role in the formation of complicated structures in the dynamics of vector fields. Here we study a specific family of systems defined on $\mathbb{R}^3$, which are invariant under the D2 symmetry group. Under the assumption that they are polynomial of degree at most two, they belong to a two-parameter family of vector fields, called the D2 model. We describe the global behavior of the system, in a region of parameter space where complicated structures occur and employ Poincaré maps to illustrate the bifurcations leading to this behavior. The existence of homoclinic orbits and heteroclinic cycles is shown, implying the presence of Shil'nikov chaos. We also explain the global bifurcations exhibited by the strange attractors as an effect of symmetry and conclude by describing the behavior of the system at infinity.

Cubic differential systems with degenerate infinity and a triplet of parallel invariant straight lines

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Consider the real cubic system of differential equations

$$\dot{x} = \sum_{j=0}^{3} P_j\left(x,y\right) \equiv P\left(x,y\right), \qquad \dot{y} = \sum_{j=0}^{3} Q_j\left(x,y\right) \equiv Q\left(x,y\right), \tag{3}$$

where P_j, Q_j are homogeneous polynomials of degree j. If $yP_3\left(x,y\right) - xQ_3\left(x,y\right) \equiv 0$ we say that the infinity is degenerated for (3) , i.e. consists only of singular points.

A straight line $\alpha x + \beta y + \gamma = 0$, $\alpha, \beta, \gamma \in \mathbb{C}$ is called invariant for system (3) if there exists a polynomial $K(x,y)$ such that the identity $\alpha P\left(x,y\right) + \beta Q\left(x,y\right) \equiv \left(\alpha x + \beta y + \gamma\right) K\left(x,y\right)$ holds.

The quadratic systems with degenerate infinity has been studied in [1]. In this paper the qualitative investigation of some cubic systems with degenerate infinity and invariant straight lines is given.

Theorem. Every cubic differential system with degenerate infinity and six invariant straight lines three of which are parallel via affine transformation and time recalling can be written as one of the following two systems:

$$\begin{cases} \dot{x} = x\,(x+1)\,(x-a) \\ \dot{y} = y\left(a + by + x^2\right), a > 0, \end{cases} \qquad \begin{cases} \dot{x} = (x-a)\left(x^2+1\right) \\ \dot{y} = y\left(-1 - 2ax + by + x^2\right) \end{cases}$$

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Section 2
Algebra and logic

About the constructions that permit to obtain the theories of relativity torsion

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On invariant Haar measure on topological quasigroups

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We study some topological properties of medial and paramedial quasigroups. We examine Haar measures on paramedial topological quasigroups with multiple identities.

Factorization structures with nonhereditary classes of projections

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In the category of the local convex topological vectorial Hausdorff spaces, we build a proper class of the factorization structures for which the classes of projections are not hereditary with respect to the class of universal monomorphisms.

About a application defined of the left product

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The authors study the application of the $\mathbb{K}(\mathcal{M}_u)$ class (the class of the $\mathcal{M}_u$-coreflective subcategories) by the left product of two subcategories into the subcategory $\mathcal{C}_2\mathcal{V}$, of topological vector locally convex spaces Hausdorff.

Three methods of constructing semireflexive subcategories

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Three methods of constructing semireflexive subcategories are given.

On homomorphism of topological groupoids with a continuous division

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L. Pontrjagin has proved that for a large class of topological groups the homomorphism mapping is open. M. Choban has generalized this assertion to topological algebras with a continuous signature. We give the conditions when continuous homomorphisms of n-topological groupoids with a continuous division are open.

On lower and upper central series of Moufang loops

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For definitions and properties of Moufang loops, see [1]. It is well known that the factors of (transfinite) upper central series and (transfinite) lower central series of any loop are abelian groups [1]. Let Q be a Moufang loop with (transfinite) upper and lower central series $\{\mathcal{Z}_\alpha(Q)\}$ and $\{\mathcal{A}_\xi(Q)\}$ respectively. The following statements are proved.

If for an integer $k \geq 0$ the factor loop $\mathcal{A}_k(Q)/\mathcal{A}_{k+1}(Q)$ has a finite exponent λ_k, then the factor loop $\mathcal{A}_{k+1}(Q)/\mathcal{A}_{k+2}(Q)$ has the finite exponent λ_k and λ_{k+1} divide λ_k.

If for a natural number $k \geq 1$ the factor loop $\mathcal{Z}_{k-1}(Q)/\mathcal{Z}_{k-2}(Q)$ has the finite exponent λ_{k-1}, then the factor loop $\mathcal{Z}_k(Q)/\mathcal{Z}_{k-1}(Q)$ has the finite exponent λ_k and λ_k is the divider of λ_{k-1}.

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Contributions of G. S. Nadiu in non-classical logic

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In 1967, Gheorghe S. Nadiu publishes in "Mathematical studies and researches", the article "On a method for the construction of Three - valued ?ukasiewicz algebras" (Romania), cited in the book "Cylindric Algebras" by P. Monk, L. Henkin, A. Tarski. This article attracted the attention of Grigore C. Moisil, who offers him a scholarship, taking him out of production to make his doctorate at the Mathematic Institute of the Romanian Academy. He publishes more than 50 scientific papers in Theory of Algorithms, Logics of mathematics and Theory of Categories fields. "Gheorghe S. Nadiu, by introducing the notion of quantified filter in a boolean monadic algebra, showed how one can obtain a trivalent Łukasiewicz algebra; also, he obtained an algebraic characterization of a completitude theorem from the intuitionist logic of Kripke." (G. St. Andonie, Science History in Romania, Academic Publishing House, SRR, 1981).

On commutative Moufang loop with centralizer satisfying some finitiness conditions

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The following concept is the natural generalization of the concept of centre. Let M be a subset and H be a subloop of commutative Moufang loop L. The set $Z_H(M) = \{x \in H | x \cdot yz = xy \cdot z, \ \forall y, z \in M\}$ is called centralizer of subset M into subloop H. $Z_H(M)$ is a subloop of L [1]. It is proved that a commutative Moufang loop satisfies one of the following properties: is finite; is finitely generated; has a finite (special) rank; maximum condition for its subloops; minimum condition for its subloops if the centralizer of one of its finitely generated subloops, satisfies this property.

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Some results on the orthogonal groupoids

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A groupoid is a pair $(Q, *)$, where Q is a set and " $*$ " a binary operation on Q (a function from $Q \times Q$ to Q). We usually write the image of the operation on the pair (a, b) as $a * b$. An operation table or Cayley table of a set with a binary operation is the square array, having rows and columns indexed by Q in some order (the same order for rows as for columns), for which the entry in row a and column b is $a * b$.

A groupoid $(Q, *)$ is a left (right) quasigroup if an only if the equation $a*x = b$ $(y * a = b)$ has a unique solution in Q for any $a, b \in Q$, that is the mapping $L_a : Q \to Q, \quad L_a(x) = a*x \quad (R_a : Q \to Q, \quad R_a(x) = x*a)$ is a permutations of Q for any $a \in Q$. Two groupoids $(Q, *)$ and $(Q, \circ)$ are said to be *orthogonal* if for any $a, b \in G$ the system of equations $x*y = a$, $x \circ y = b$ has a unique solution in Q which is equivalent to the fact that the mapping $(x, y) \mapsto (x * y, x \circ y)$ is a permutation of $Q \times Q$. In this case the groupoid $(Q, \circ)$ is called a orthogonal complement for $(Q, *)$. Orthogonality is a symmetric property. It is rather easy to construct a pair $(Q, *), (Q, \circ)$ of orthogonal groupoids on the set Q. We start from a arbitrary permutation φ of $Q \times Q$ and for any $x, y \in Q$ put $x * y = u, \quad x \circ y = v$ if and only if $\varphi(x, y) = (u, v)$.

Denote $Q_{\underset{a}{\cdot}} = \{(x, y) \in Q \times Q | x \cdot y = a\}$.

Proposition 1. *i) If a groupoid $(Q, \cdot)$ has an orthogonal complement than $Q \cdot Q = Q$, where $Q \cdot Q = \{a \cdot b | a, b \in Q\}$;*

ii) A finite groupoid $(Q,\cdot)$ has an orthogonal complement if and only if $|Q_{\dot{a}}| = |Q|$ for all $a \in Q$;

iii) If $|Q| = n$, and a groupoid $(Q,\cdot)$ satisfy ii) than it has exactly $(n!)^n$ orthogonal complements.

Proposition 2. *Let be $Q \neq \oslash$ and $(Q,\cdot)$ be a groupoid. Define groupoids $(Q,*_1),(Q,*_2)$ by: $x *_1 y = x \cdot xy$; $\quad x *_2 y = xy \cdot x$ for all x, $y \in Q$.*

i) If $(Q,\cdot)$ and $(Q,*_1)$ are orthogonal, then $(Q,\cdot)$ is a groupoid with right division, that is the equation $y \cdot a = b$ has a solution for any a, $b \in Q$;

ii) A finite groupoid $(Q,\cdot)$ is a quasigroup if and only if it is orthogonal to $(Q,*_1)$;

iii) A groupoid $(Q,\cdot)$ is a left quasigroup if and only if it is orthogonal to $(Q,*_2)$.

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The relations between the absence of finite approximation relative to model-completeness in the provability-intuitionistic logic and the number of model-pre-complete classes

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We build propositional logic's formulas from the variables $p, q, r, \ldots$, using the operators $\&, \vee, \supset \neg, \Delta$ and parentheses. The provability-intuitionistic calculus I^{Δ} [1] is based on the notion of formula and it is defined by the intuitionist calculus, enriched with three Δ-axioms: $(p \supset \Delta p)$, $((\Delta p \supset p) \supset p)$, $((p \supset q) \supset p) \supset (\Delta q \supset p)$. As the logic of a calculus we understand the set of all formulas deductible in this calculus. A set of formulas, that contain the axioms of the logic I^{Δ}, and which is closed relative to the inference rules of this calculus is called the *extension* of the logic I^{Δ}. In this paper we consider the logic LC^{Δ} that is defined as $I^{\Delta} + ((p \supset q) \vee (q \supset p))$.

Suppose the logic L satisfies conditions $I^{\Delta} \subseteq L \subseteq LC^{\Delta}$.

A formula $F(p_1, \ldots, p_n)$ of the logic L is named a *model* for boolean function $f(p_1, \ldots, p_n)$, if the identity $F(p_1, \ldots, p_n) = f(p_1, \ldots, p_n)$ is true on the set $\{0; 1\}$. The system Σ of formulas of the logic L is said to be *model-complete* in L, if, for any boolean function, at least one of its model is expressible in L by Σ, i.e. it can be obtained from variables and formulas of Σ by means of weak substitution rule and replacement rule by equivalent in L. The system Σ is called *model-pre-complete* in L, if Σ is not model-complete in L but for any formula F, which is not expressible in L by Σ, the system $\Sigma \bigcup \{F\}$ is model-complete in L.

A logic L is called finitely approximated relative to model-completeness, if for any finite system of formulas Σ, which is not model-complete in L, there is a tabular extension of the logic L, which also is not model-complete.
Theorem. *Logic I^{Δ}, logic LC^{Δ}, and all logics intermediate between them are not finitely approximated relative to model-completeness.*
Corollary. *For the logics, I^{Δ}, LC^{Δ}, as well as for any intermediate logic between them, there is no criterion of model completeness, traditionally formulated in terms of a finite number of model pre-complete classes.*
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Some theoretical aspects of spatial groups of generalized symmetry

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The geometrical component g of $\bar{P}$-symmetry [1] or W_q-symmetry [2] transformations $g^(p)$ or $g^(w)$ acts on the points of the geometrical figure and on the "physical" indexes-quality, located in these points, according to one independent determinate law. The classical space groups of symmetry (periodic or semi periodic) have the elements that contain parallel translations. In the generalization of these groups with $\bar{P}$-symmetry, with W_p-symmetry [3,1] or W_q-symmetry there are some specific theoretical aspects. To develop the theory of generalized symmetry groups of discrete space groups generating these features call for the development of algebraic device. In particular, we study the properties of standard Cartesian wreath product and the properties of crossed standard Cartesian wreath product of a finite group with a periodic or semi periodic group.
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On groups of automorphisms of commutative Moufang loops

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For definitions and properties of Moufang loops, see [1]. In the present paper the following result is proved.

Theorem 1. *The following properties of the CML Q with multiplication group $\mathfrak{M}$ are equivalent:*

1) *Q satisfies the minimum condition for subloops;*

2) a) *Q is a torsion loop,*

b) *every torsion group of automorphisms of Q is finite;*

3) a) *Q is a torsion loop,*

b) *every torsion group of automorphisms of Q satisfies the minimum condition for subgroups;*

4) a) *Q is a torsion loop,*

b) *Q satisfies one of the equivalent conditions: i) the group of inner mappings of CML Q is a finite 3-group; ii) the associator loop of CML Q is a finite 3-loop; iii) the quotient loop $Q/Z(Q)$, where $Z(Q)$ is center of CML Q, is an elementary finite 3-loop; iv) there exists an associative normal subloop H of CML Q such that Q/H and (H, Q, Q) are finite 3-loops;*

5) a) *Q is a torsion loop,*

b) *$\mathfrak{M}$ admits matrix representation over a certain field of characteristic zero;*

6) a) *Q is a torsion loop,*

b) *normal (or non-abelian) subgroups of $\mathfrak{M}$ admit matrix representation over a certain field of characteristic zero;*

7) a) *Q is a torsion loop,*

b) *at least one maximal abelian subgroup of $\mathfrak{M}$ admits matrix representation over a certain field of characteristic zero;*

8) a) *Q is a torsion loop,*

b) *if $\mathfrak{M}$ contains a nilpotent (or solvable) subgroup of class n, then all nilpotent (or solvable) subgroups of class n admit matrix representation over a certain field of characteristic zero;*

9) a) *Q is a torsion loop,*

b) *every torsion group of automorphisms of $\mathfrak{M}$ is finite;*

10) a) *Q is a torsion loop,*

b) *$\mathfrak{M}$ satisfies one of the equivalent conditions: i) the group of inner automorphisms of $\mathfrak{M}$ is a finite 3-group; ii) the quotient group $\mathfrak{M}/Z(\mathfrak{M})$, where $Z(\mathfrak{M})$ is center of group $\mathfrak{M}$, is a finite 3-group; iii) the commutator group of group $\mathfrak{M}$ is a finite 3-group; iv) there exists an abelian normal subgroup $\mathfrak{N}$ of group $\mathfrak{M}$ such that $\mathfrak{M}/\mathfrak{N}$ and $(\mathfrak{N}, \mathfrak{M})$ are finite 3-groups,*

c) *primary elementary abelian groups of automorphisms of $\mathfrak{M}$ are countable.*

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Some Results on Prime Fuzzy Subhypermodules

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Let R be a commutative hyperring with identity and M be an unitary R-hypermodule. We introduce and characterize the prime fuzzy subhypermodules of M. We investigate the Zariski topology on FHspec(M), the prime fuzzy spectrum of M, the collection of all prime fuzzy subhypermodules of M.

A short survey on the spectrum of prime fuzzy submodules

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In this paper we introduce and characterize the notion of prime fuzzy submodules of a unitary module over a commutative ring with identity. We topologize FSpec(M), the collection of all prime fuzzy submodules of M, analogous to that for FSpec(R), the spectrum of fuzzy prime ideals of R, and investigate the properties of this topological space. In particular, we will study the relationship between FSpec(M) and FSpec(R/Ann(M)) and obtain some results.

Criterion for functional completeness in the 16 element algebraic model of pre-tabular model of em4 logic

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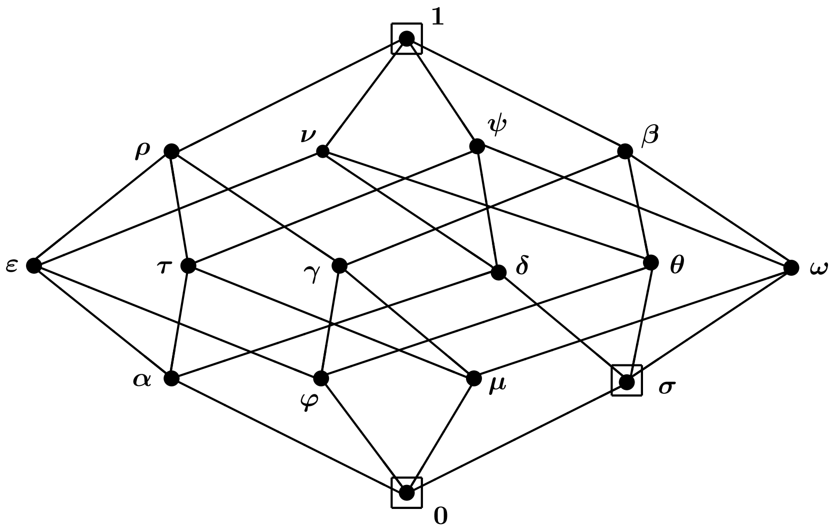
In 1975 in the works of L. Esakia and V. Meshi [1] and independent of L.Maksimova [2] the pre-tabular modal logic $EM4$ was discovered. It is approximated by the logics of a series of topological Boolean algebras $\Delta_i (i = 1, 2, ...)$ of order 2^i with 3 open elements. The fourth, i.e. Δ_4 of these algebras we represent by of the following diagram:

The logic of this algebra represents the extension of the logic $S4$, generated by three formulas as a new axioms pointed out in [1, 2]. M.Coban and author [3] obtained the criterions for functional completeness in the $4-$vulued and in the



8-valued extensions of $EM4$ in the modal logic. In the present communication we give aut analogous criterion for the 16−valued extension of $EM4$ modal logic. Let denote by symbols M_1, M_2 and M_3 the following matrixes:

$$(0\alpha\varphi\mu\gamma\tau\rho\sigma\theta\delta\nu\psi\beta 1) \quad \begin{pmatrix} 0\alpha\mu\gamma\varepsilon\rho\sigma\omega\nu\delta\beta 1 \\ 0\alpha\varepsilon\gamma\mu\rho\sigma\nu\omega\delta\beta 1 \end{pmatrix} \quad \begin{pmatrix} 0\alpha\varphi\mu\varepsilon\tau\gamma\rho\sigma\delta\theta\omega\nu\psi\beta 1 \\ 0\rho\rho\rho\rho\rho\rho\rho\sigma\sigma\sigma\sigma\sigma\sigma\sigma 1 \end{pmatrix}$$

Theorem. *In order that a system of formulas Σ to be functionally complete in the 16−element extension of $EM4$ logic it is necessary and sufficient that Σ to be functionally complete in the 8−element extension of considered logic and for each of 3 indicated matrices to exists in Σ a formula that does not preserve this matrix.*

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Prime Submodules of Gamma Modules

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Let Γ be an abelian group and R be a Γ- ring. Let M be a Γ module over R. In this framework we investigate some basic properties of prime Γ submodules.

On functional expressibility of systems of formulas containing paraconsistent negation

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An extension of a paraconsistent logic is considered. Various types of expressibility of formulas in that logic are examined, too. Connected with the notion of expressibility we consider the problem of determining the complete with respect to expressibility sets of formulas in it. Some necessary and sufficient conditions for the above mentioned problem are obtained.

Baer-invariant of some non finitely generated abelian groups

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We proved that the Baer-invariant of Q is always trivial and also Baer-invariant of abelian groups Q/Z and $Z(p^\infty)$, with respect to the varieties of polynilpotent, nilpotent and outer commutator in some condition $[N_{c_1}; N_{c_2}]$ are trivial. Then after computing the Baer-invariant of Z_n with respect to Burnside variety, we have concluded it for Q/Z Burnside variety.

On structure of Moufang loops

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The structure of Moufang loops is investigated with the help of alternative algebras, particularly, the Jevlakov radical, the multiplicative loop of invertible elements, the circle loop of quasi-regular elements. Let $\mathcal{M}$ denote the class of all Moufang loops and let $\mathcal{S}$ denote the class of such Moufang loops without non-associative subloops, which are simple loops. It is proved that $\mathcal{S}$ is a hereditary radical of class $\mathcal{M}$.

Let $\mathcal{P}$ be the semi-simple class of radical $\mathcal{S}$. Then $Q/\mathcal{S}(Q) = \mathcal{P}(Q)$ for any loop $Q \in \mathcal{M}$, where $\mathcal{S}(Q) \in \mathcal{S}$, $\mathcal{S}(P) \in \mathcal{P}$. It is proved that the loop $\mathcal{P}(Q)$

decomposes into a direct product of finite number of non-associative simple Moufang loops. The non-associative simple Moufang loops are described in [1].

The paper also proves that the loops of radical $\mathcal{S}$ can be imbedded into a multiplicative loop of invertible elements of alternative algebras, which are a homomorphic image of loop algebra. The other Moufang loops, i.e. such Moufang loops Q that $\mathcal{P}(Q) \neq 1$, are not embedded into a loop of invertible elements of any alternative algebra. This answers the known question of theory of Moufang loop. See, for example, [2-4].

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New applications of the algebraic hyperstructures

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Some applications of hyperrings and hyperfields in geometry and in number theory are presented.

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Free orderable A-loops

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An algebra L with an operation of multiplication and two operations of division /, \ in which the identities

$$x(x\backslash y) = x/(xy) = (y/x)x = (yx)/x = y, x/x = y\backslash y$$

are true is called a *loop*. Considering $e = y\backslash y$, we get $ex = xe = x$ for every element $x \in L$. So the element e is the unite the loop L. The group of substitutions of the loop L generated by all the translations of the form L_x or R_y, where $L_x y = R_y x = xy$ $(x, y \in L)$ is called a *multiplication group* of the loop L. A permutation α of the multiplication group is called *inner* if $\alpha(e) = e$. A loop L with a binary predicate $\leq$ where the quasiidentities

$$x \leq x, \;\; x \leq y \;\&\; y \leq x \Rightarrow \; x = y, \;\; x \leq y \;\&\; y \leq z \Rightarrow \; x \leq z,$$
$$x \leq y \Rightarrow t(xz) \; \leq t(yz), \quad x \leq y \Rightarrow t\backslash(x/z) \; \leq t\backslash(y/z)$$

are true is called *partially ordered.* If for every pair of elements x, y of the partially ordered loop L $x \leq y$ or $y \leq x$, then L is called a *linearly ordered loop.* We say that a loop L is free orderable if any binary relation $\leq$ which is defined in L for some pairs of elements, in respect to which L is partially ordered, can be defined also for the rest pairs of elements so that L be linearly ordered. Concepts given above can be find in [1]–[3]. A loop L is called an *A-loop* if all its inner permutations are automorphisms (see [4]). It is proved: *Any nilpotent finitely generated A-loop without torsions is free orderable.* From this, according to local theorem, *any locally nilpotent A-loop without torsions is free orderable.*

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Section 3
Topology and differential geometry

On special external bases of countable order of topological spaces

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One of the important problems in general topology is to establish concrete connections between various classes of spaces in terms of continuous mappings satisfying certain additional conditions. There is a long and fruitful tradition in general topology to use natural restrictions on bases and to characterize or to introduce various important classes of spaces. In particular, this was done by J. Nagata, R. Bing and Ju. Smirnov (see [10]). The work in this direction has lead to discovery of important kinds of bases. In particular, we introduce some new kinds of sharp bases and of bases of countable order. Some open questions are formulated.

We use the terminology from [5, 10, 9]. Any space we consider is assumed to be a T_1-space, $\omega = \{0, 1, 2, ...\}$, $S = \{0\} \cup \{2^{-n} : n \in \omega\}$ and $I = [0, 1]$ are subspaces of the space of reals.

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Integral properties of certain class of analytic functions with negative coefficients

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In this paper integral properties of certain class of analytic functions with negative coefficients are studied. This class is defined using a generalized Sălăgean operator. The obtained results are sharp and they improve known results.

On the compactifications of E-metric spaces

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Any space is considered to be Tychonoff and non-empty. We use the terminology from [2].

In [1] there were introduced the metric spaces over topological semifields. The notion of a topological semifield may be generalized in the following way. We say that E is *an m-scale* if: E is a topological algebra over the field of reals $\mathbb{R}$; E is a commutative ring with the unit $1 \neq 0$; E is a vector lattice and $0 \leq xy$ provided $0 \leq x$ and $0 \leq y$; for any neighborhood U of 0 in E there exists a neighborhood V of 0 in E such that if $x \in V$ and $0 \leq y \leq x$, then $y \in U$; for any non-empty upper bounded set A of E there exists the supremum $\vee A$.

Let E be an m-scale. Denote by $E^{-1} = \{x : x \cdot y = 1 \textit{ for some } y \in E\}$ the set of all invertible elements of E. By $N(0, E)$ we denote some base of the space E at the point 0. We consider that $0 \leq x \ll 1$ if $0 \leq x < 1$, $1 - x$ is invertible and $lim_{n \to \infty} x^n = 0$. We put $E^{(+,1)} = \{x \in E : 0 \leq x \ll 1\}$.

A mapping $d : X \times X \longrightarrow E$ is called *a pseudo-metric over m-scale E* or *a pseudo-E-metric* if it is satisfying the following axioms: $d(x, x) = 0$; $d(x, y) = d(y, x)$; $d(x, y) \leq d(x, z) + d(yz, y)$.

Every pseudo-E-metric is non-negative. A pseudo-E-metric d is called an E-metric if it is satisfying the following axiom: $d(x, y) = 0$ if and only if $x = y$.

The ordered triple (X, d, E) is called *a metric space over the m-scale E* or *an E-metric* space if d is an E-metric on X.

Fix an E-metric space (X, d, E). For any point $x \in X$, any subset $L \subseteq X$ and any $U \in N(0, E)$ we put $B(x, d, U) = \{y \in X : d(x, y) \in U\}$ and $B(L, d, U) =$

$\cup\{B(x,d,U) : x \in L\}$. The family $\{B(x,d,U) : x \in X, U \in N(0,E)\}$ is the base of the topology $\mathcal{T}(d)$ of the pseudo-E-metric space (X,d,E). If d is an E-metric, then the space $(X,\mathcal{T}(d))$ is a Tychonoff (completely regular and Hausdorff) space. The E-metric d generated the uniformity $\mathcal{U}(d)$ and the proximity δ_d on X.

A family ξ of subsets of the space X is called d-centered (see [3] for metric spaces) if $\cap\{B(L,d,U) : L \in \eta\} \neq \emptyset$ for any $U \in N(0,E)$ and any finite subfamily $\eta \subseteq \xi$. Denote by $\beta_d X$ the family of all maximal d-centered families of subsets of the space X in the topology generated by the closed base $\{C_L = \{\xi \in \beta_d X : L \in \xi\} : L \subseteq X\}$. There exists a natural embedding $\psi : X \longrightarrow \beta_d X$, where $x \in \psi(x)$ for any $x \in X$.

Theorem 1. *$\beta_d X$ is the maximal compactification of the space $(X,\mathcal{T}(d))$ with the property: the bounded function $f : X \to \mathbb{R}$ has a continuous extension $\beta_d f : \beta_d X \to \mathbb{R}$ if and only if f is uniformly continuous.*

For any uniform space the Samuel compactification is determined [2, 4].

Corollary 1. *$\beta_d X$ is the Samuel compactification of the uniform space $(X,\mathcal{U}(d))$.*

For any proximity space the Smirnov compactification is determined [2, 5].

Corollary 2. *$\beta_d X$ is the Smirnov compactification of the proximity space (X,δ_d).*

Theorem 1 for metric spaces was proved in [3].

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About symmetry of the elements of symmetry on hyperbolic manifolds

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Geometric criterion of normal subgroup for crystallographic groups of universal covering space was extended to the factor space and their fundamental groups. Some results of mathematical crystallography for constant curvature spaces are considered [1] for manifolds too.

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On ωs-regular spaces

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The purpose of the present paper is to investigate and study the axiom of ωs-regularity which is weaker than the regularity, stronger than s-regularity and it is independent of ω-regularity. Some properties of ω-semi open sets are obtained.

On selection principles

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1. Introduction. All considered spaces are assumed to be T_1-spaces. Our terminology comes, as a rule, from [6, 13]. We continue the investigations begun in [1, 2, 11, 12, 3, 4, 5, 14].

The cardinal number $l(X) = min\{m : \textit{every open cover of } X \textit{ has an open refinement of cardinality} \leq m\}$ is the *Lindelöf number* of X.

The cardinal number $k(X) = min\{m : \textit{every open cover of } X \textit{ has an open refinement of cardinality} < m\}$ is *the degree of compactness* of X.

Let X and Y be non-empty topological spaces. A *set-valued mapping* $\theta : X \to Y$ assigns to every $x \in X$ a non-empty subset $\theta(x)$ of Y. If $\phi,\ \psi : X \to Y$ are set-valued mappings and $\phi(x) \subseteq \psi(x)$ for every $x \in X$, then ϕ is called *a selection* of ψ.

Let $\theta : X \to Y$ be a set-valued mapping and let $A \subseteq X$ and $B \subseteq Y$. The set $\theta^{-1}(B) = \{x \in X : \theta(x) \bigcap B \neq \emptyset\}$ is *the inverse image* of the set B, $\theta(A) = \theta^1(A) = \bigcup\{\theta(x) : x \in A\}$ is *the image* of the set A and $\theta^{n+1}(A) = \theta(\theta^{-1}(\theta^n(A)))$ is *the* $n+1$*-image* of the set A. The set $\theta^\infty(A) = \bigcup\{\theta^n(A) : n \in \mathbb{N}\}$ is *the largest image* of the set A.

A set-valued mapping $\theta : X \to Y$ is called *lower (upper) semi-continuous* if for every open (closed) subset H of Y the set $\theta^{-1}(H)$ is open (closed) in X.

A set-valued mapping $\theta : X \to Y$ is called perfect if it is an upper semi-continuous compact-valued mapping, $\theta^{-1}(y)$ is compact for every $y \in Y$ and $\theta : X \to \theta(X)$ is a closed mapping.

For every finite-dimensional (infinite-dimensional) normal space X there exists a single-valued continuous mapping $\theta : X \to Y$ into a finite-dimensional (infinite-dimensional) separable metric space Y such that for every single-valued factorization $(Z,\ g,\ \phi)$ it follows that $dimZ \geq dimX$. This fact is not true for set-valued lower (neither for upper) semi-continuous factorizations.

Denote by $ord(\phi) = sup\{\tau : \tau < |\phi(x)|\ for\ some\ x \in X\}$ the order of the set-valued mapping $\phi : X \to Y$. For every normal space X and any natural number $n \leq dimX$ there exist a lower semi-continuous mapping $\varphi : X \to Y$ and an upper semi-continuous mapping $\psi : X \to Y$ into a finite discrete space Y such that ψ is a selection for φ and for every lower or upper semi-continuous factorization $(Z,\ g,\ \phi)$ of φ it follows $n \leq ord(\phi)$ (see [11, 2, 12]).

Let τ be an infinite cardinal number. A mapping $\theta : X \to Y$ is called τ-dense if $l(\theta(\theta^{-1}(H))) \leq \tau$ for every $H \subseteq Y$ such that $l(H) \leq \tau$.

There is a very long list of classes of topological spaces characterized in terms of special selections of lower set-valued mappings into complete metrizable spaces (see, for example, [9, 10, 2, 11, 12, 13, 3, 4, 5, 7, 14]).

2. On paracompact p-spaces. A space X is called a paracompact p-space if it is Hausdorff and there exists a perfect single-valued mapping $\theta : X \to Y$ onto some metrizable space Y.

Theorem 1. *For a T_0-space X the following assertions are equivalent:*

1. X is a paracompact p-space;

2. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable space Z, a continuous single-valued mapping $g : Z \to Y$, a lower semi-continuous compact-valued mapping $\phi : X \to Z$ and a perfect set-valued mapping $\psi : X \to Z$ such that ϕ and ψ are τ-dense mappings for every infinite cardinal number τ, $\phi(x) \subseteq \psi(x)$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

3. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable zero-dimensional space Z, a continuous single-valued mapping $g : Z \to Y$, a lower semi-continuous compact-valued mapping $\phi : X \to Z$ and a perfect set-valued mapping $\psi : X \to Z$ such that ϕ and ψ are τ-dense mappings for every infinite cardinal number τ, $\phi(x) \subseteq \psi(x)$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

3. On strongly paracompact spaces: Let $\mathcal{P}$ be a property of topological spaces. The space X has property $loc\mathcal{P}$ if for any point $x \in X$ there exists an open subset U of X such that $x \in U$ and U has the property $\mathcal{P}$. In particular, $locl(X) \leq \tau$ if there exists an open cover γ of X such that $l(U) \leq \tau$ for any $U \in \gamma$.

Theorem 2. *For a T_0-space X the following are equivalent:*

1. X is strongly paracompact;

2. For every cardinal number τ and a lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y with $locl(Y) \leq \tau$ there exist a complete metrizable space Z, an open continuous single-valued mapping $g : Z \to Y$ onto Y, a lower semi-continuous τ-dense mapping $\phi : X \to Z$ and an upper semi-continuous compact-valued τ-dense mapping $\psi : X \to Z$ such that $locl(Z) \leq \tau$, $\phi(x) \subseteq \psi(x)$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

3. For every cardinal number τ and a lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y with $locl(Y) \leq \tau$ there exist a zero-dimensional complete metrizable space Z, an open continuous single-valued mapping $g : Z \to Y$ onto Y, a lower semi-continuous τ-dense mapping $\phi : X \to Z$ and an upper semi-continuous compact-valued τ-dense mapping $\psi : X \to Z$ such that $locl(Z) \leq \tau$, $\phi(x) \subseteq \psi(x)$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

4. The space X is regular and for every cardinal number τ and a lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y with $locl(Y) \leq \tau$ there exist a complete metrizable space Z, an open continuous single-valued mapping $g : Z \to Y$ onto Y, a lower semi-continuous τ-dense mapping $\phi : X \to Z$ such that $locl(Z) \leq \tau$ and $g(\phi(x)) \subseteq \theta(x)$ for every $x \in X$.

5. For every cardinal number τ and a lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y with $locl(Y) \leq \tau$ there exist a complete metrizable space Z, an open continuous single-valued mapping $g : Z \to Y$ onto Y an upper semi-continuous τ-dense mapping $\psi : X \to Z$ such that $locl(Z) \leq \tau$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

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On factorization principles

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1. Introduction. All considered spaces are assumed to be T_1-spaces. Our terminology comes, as a rule, from [1, 5]. We continue the investigations begun in [3, 4].

The cardinal number $l(X) = min\{m : every\ open\ cover\ of\ X\ has\ an\ open\ refinement\ of\ cardinality \leq m\}$ is the *Lindelöf number* of X.

Denote by τ^+ the least cardinal number larger than the cardinal number τ. It is obvious that $l(X) \leq k(X) \leq l(X)^+$.

Let X and Y be non-empty topological spaces. A *set-valued mapping* $\theta : X \to Y$ assigns to every $x \in X$ a non-empty subset $\theta(x)$ of Y. If $\phi,\ \psi : X \to Y$ are set-valued mappings and $\phi(x) \subseteq \psi(x)$ for every $x \in X$, then ϕ is called *a selection* of ψ.

Let $\theta : X \to Y$ be a set-valued mapping and let $A \subseteq X$ and $B \subseteq Y$. The set $\theta^{-1}(B) = \{x \in X : \theta(x) \bigcap B \neq \emptyset\}$ is *the inverse image* of the set B, $\theta(A) = \theta^1(A) = \bigcup\{\theta(x) : x \in A\}$ is *the image* of the set A and $\theta^{n+1}(A) = \theta(\theta^{-1}(\theta^n(A)))$ is *the* $n+1$*-image* of the set A. The set $\theta^\infty(A) = \bigcup\{\theta^n(A) : n \in \mathbb{N}\}$ is *the largest image* of the set A.

A set-valued mapping $\theta : X \to Y$ is called *lower (upper) semi-continuous* if for every open (closed) subset H of Y the set $\theta^{-1}(H)$ is open (closed) in X.

Let X and Y be topological spaces and $\theta : X \to Y$ be a set-valued mapping. A factorization for θ is a triple $(Z,\ g,\ \phi)$, where $g : Z \to Y$ is a continuous single-valued mapping from the space Z into Y and $\phi : X \to Z$ is a set-valued mapping such that $g(\phi(x)) \subseteq \theta(x)$ for every $x \in X$. If ϕ is a single-valued continuous mapping (respectively, lower or upper semi-continuous), then the factorization $(Z,\ g,\ \phi)$ is called single-valued (respectively, lower or upper semi-continuous) factorization. Note that every factorization $(Z,\ g,\ \phi)$ generates a selection $g \circ \phi : X \to Y$ for θ. The concept of the factorization of the single-valued mappings was introduced by S.Mardešić in [2]. The case of set-valued mappings was examined in [3, 4].

For every finite-dimensional (infinite-dimensional) normal space X there exists a single-valued continuous mapping $\theta : X \to Y$ into a finite-dimensional (infinite-dimensional) separable metric space Y such that for every single-valued factorization $(Z,\ g,\ \phi)$ it follows that $dimZ \geq dimX$. This fact is not true for set-valued lower (neither for upper) semi-continuous factorizations.

Denote by $ord(\phi) = sup\{\tau : \tau < |\phi(x)|\ for\ some\ x \in X\}$ the order of the set-valued mapping $\phi : X \to Y$. For every normal space X and any natural number $n \leq dimX$ there exist a lower semi-continuous mapping $\varphi : X \to Y$ and an upper semi-continuous mapping $\psi : X \to Y$ into a finite discrete space

Y such that ψ is a selection for φ and for every lower or upper semi-continuous factorization $(Z,\ g,\ \phi)$ of φ it follows $n \leq ord(\phi)$ (see [3, 4]). Moreover, a regular space X is extremally disconnected if and only if any upper semi-continuous compact-valued mapping $\theta : X \to Y$ has a single-valued factorization.

2. Factorization principles. A set-valued mapping with a property Q is called a Q-mapping. A property Q of set-valued mappings is called a perfect property if for every set-valued Q-mapping $\theta : X \to Y$ from a space X into a metrizable space Y, every perfect mapping $h : Z \to Y$ from a metrizable space Z onto Y and every subspace H of Z, such that $h(H) = Y$, the mapping $\phi : X \to Z$, where $\phi(x) = cl_Z(H \cap h^{-1}(\theta(x)))$ for ever $x \in X$, is a Q-mapping.

Let τ be an infinite cardinal number. A mapping $\theta : X \to Y$ is called τ-dense if $l(\theta(\theta^{-1}(H))) \leq \tau$ for every $H \subseteq Y$ such that $l(H) \leq \tau$.

The property of a set-valued mapping to be τ-dense is a perfect property.

Theorem 1 (U-factorization principle). *For a space X and a perfect property Q of set-valued mappings the following assertions are equivalent:*

1. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable space Z, a continuous single-valued mapping $g : Z \to Y$ and an upper semi-continuous mapping $\psi : X \to Z$ with the property Q such that $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

2. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable zero-dimensional space Z, a continuous single-valued mapping $g : Z \to Y$ and an upper semi-continuous mapping $\psi : X \to Z$ with the property Q such that $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

Theorem 2 (L-factorization principle). *For a space X and a perfect property Q of set-valued mappings the following assertions are equivalent:*

1. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable space Z, a continuous single-valued mapping $g : Z \to Y$ and a lower semi-continuous mapping $\phi : X \to Z$ with the property Q such that $g(\phi(x)) \subseteq \theta(x)$ for every $x \in X$.

2. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable zero-dimensional space Z, a continuous single-valued mapping $g : Z \to Y$ and a lower semi-continuous mapping $\phi : X \to Z$ with the property Q such that $g(\phi(x)) \subseteq \theta(x)$ for every $x \in X$.

Theorem 3 (M-factorization principle). *For a space X and a perfect property Q of set-valued mappings the following assertions are equivalent:*

1. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable space Z, a continuous single-valued mapping $g : Z \to Y$, a lower semi-continuous mapping $\phi : X \to Z$ and an upper semi-continuous mapping $\psi : X \to Z$ both ϕ and ψ with the property Q such that $\phi(x) \subseteq \psi(x)$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

2. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable zero-dimensional space Z, a continuous single-valued mapping $g : Z \to Y$, a lower semi-continuous mapping $\phi : X \to Z$ and an upper semi-continuous mapping $\psi : X \to Z$ with the property Q such that $\phi(x) \subseteq \psi(x)$ and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

Corollary 4. *For a T_0-space X the following assertions are equivalent:*

1. X is a Hausdorff paracompact space;

2. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist an upper semi-continuous mapping $\psi : X \to Y$ such that $\psi(x) \subseteq \theta(x)$ for every $x \in X$.

3. For every lower semi-continuous mapping $\theta : X \to Y$ into a complete metrizable Y there exist a complete metrizable zero-dimensional space Z, a continuous single-valued mapping $g : Z \to Y$, a lower semi-continuous compact-valued mapping $\phi : X \to Z$ and an upper semi-continuous compact-valued mapping $\psi : X \to Z$ such that $\phi(x) \subseteq \psi(x)$, $\psi^{\infty}(x)$ is a separable space and $g(\psi(x)) \subseteq \theta(x)$ for every $x \in X$.

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On classes of generalized quasiconformal mappings

Victoria Stanciu

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We present normality criteria for families of Q-homeomorphisms (ring Q-homeomorphisms) with Q in the function classes BMO, FMO, FLD.

Section 4
Functional analysis and differential equations

Carleman Estimates for the Magnetohydrodynamic Equations on the Torus

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This paper proves a Carleman estimate for the adjoint magnetohydrodynamic equations, which describe the dynamics of electrically conducting fluids, with periodic conditions taken into account. The main importance of this result consists in the establishment of the observability inequality for the above mentioned equations and, further, in proving the local exact controllability for the magnetohydrodynamic equations on the torus.

On a stable finite element method for the equations of linear elasticity

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Stable finite element methods are used in order to obtain a spatial approximation with equal-order polynomials for different variables of the problem. A such method, used for the Stokes equations, is extended for the equations of linear elasticity.

Convergence in fuzzy measure for measurable multifunctions

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The goal of this talk is to outline some properties of sequences of measurable multifunctions that are convergent in fuzzy measure. We use a set-norm (introduced in [1]) on the family of nonempty subsets of a real linear space.

Predictor-Corrector Method for finding the numerical solution of the initial value problem of the ordinary differential equation with singular point

Kanoknapa Erawun, Pimpak Phataranavik, Huachiew Chalermprakiet

Roi Et Rajabhat University, Thailand

In [1], M. Podisuk, U. Chandang and W. Sanprasert introduced the multi-step integration method to find the numerical solution of the initial value problem of the ordinary differential equation. W. Sanprasert, U. Chandang and M. Podisuk, in [2], used the integration method with orthogonal polynomials to find the numerical solution of the initial value problem of ordinary differential equation with singular point. K. Erawun, in [3], used the multi-step formulas in [2] together with new multi-step integration formulas as the explicit method to find the approximate solution of the problem in [2]. In this paper, we use the multi-step integration formulas to find the problem in [2] and [3] together with some new multi-step integration formulas. The results by computer programming will be illustrated.

On band preserving operators on Banach lattices

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In this talk, we are interested in the band preserving operators on Banach lattices. For example, let X be a Banach lattice and let T, T' be bijective band preserving operator on X, X', respectively. Then, inverse of T is also a band preserving operator.

Smoothness properties of semigroups for abstract elliptic operators

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The paper consists mainly in the following statements.

Lemma. 1° If $D(B)$ is dense in E and the operator B has regular points, then the domain $D(B^p)$ is dense in E for any $p = 2, 3, \ldots$.
2° For $x_0 \in D(B^p)$, the solution $U(t)x_0$ of the equation

$$\dot{x} = Bx, \quad x(0) = x_0 \tag{4}$$

for $t \geq 0$ has $p-1$ continuous derivatives and the derivative of the order p, continuous for $t > 0$.

Theorem. Let Γ_α be the boundary $\partial\Omega(\alpha)$ of the domain $\Omega(\alpha)$ and B be abstract elliptic operator which is the generator of strongly continuous semigroup $U(t)$. Then for $t > 0$ the semigroup $U(t)$ is $p = 1, 2, 3, ...$ times continuously differentiable and for any $x_0 \in D(B^p)$

$$U^{(p)}(t)x_0 = B^p U(t)x_0 = -\frac{1}{2\pi i}\int\limits_{\Gamma_\alpha} \lambda^p e^{\lambda t} R_\lambda(B) x_0 d\lambda, \tag{5}$$

for $t > 0$ the following estimate

$$\|U^{(p)}(t)\| \leq M e^{\alpha t} t^{-p} \tag{6}$$

is true, and $x^{(p)}(0) = B^p x_0$.

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Potentiality conditions of Lyapounov-Schmidt branching equation at Poincaré-Andronov-Hopf bifurcation

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In the extension of the results on potentiality of Lyapounov-Schmidt branching equation (BEq) for stationary bifurcation, published by V.A.Trenogin and N.A.Sidorov, in Uzbek. Math. J., **2**(1992), 40-49, the potentiality conditions of the BEq at Poincaré-Andronov-Hopf bifurcation are searched. By using Poincaré's change of variables this problem is reduced to the analogous one in complexified Banach spaces. Then, with certain assumptions, necessary and sufficient conditions of Lyapounov-Schmidt BEq for nonstationary bifurcation are established.

On a nonlinear second-order multi-point boundary value problem

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We study the existence and nonexistence of positive solutions of the nonlinear second-order differential system

$$(S) \qquad \begin{cases} u''(t) + c(t)f(v(t)) = 0, & t \in (0,T), \\ v''(t) + d(t)g(u(t)) = 0, & t \in (0,T), \end{cases}$$

with the multi-point boundary conditions

$$(BC) \qquad \begin{cases} \alpha u(0) - \beta u'(0) = 0, \;\; u(T) = \displaystyle\sum_{i=1}^{m-2} a_i u(\xi_i) + a_0, \;\; m \geq 3, \\ \gamma v(0) - \delta v'(0) = 0, \;\; v(T) = \displaystyle\sum_{i=1}^{n-2} b_i v(\eta_i) + b_0, \;\; n \geq 3. \end{cases}$$

The proof of the existence of solutions are based on the Schauder fixed point theorem and some auxiliary results.

Existence of three solutions for a class of fractional boundary value problems

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In this work, by employing the Leggett-Williams fixed point theorem, we study the existence of at least three positive solutions of boundary value problems for a system of third-order ordinary differential equations with (p,q)-Laplacian.

On the equilibria of generalized dynamical systems

Vasile Postolică

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This research work contains original properties for the equilibria (ideal or critical) points sets including important generalized dynamical systems, in strong connections with the Vector Optimization by the Efficiency and the Potential Theory following the Choquet's Boundaries.

On certain algebraic objects in the study of spectral properties of linear relations

Adrian Sandovici

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The main ingredients of this talk are the ascent, descent, nullity and defect of a linear relation in a Banach space. Their algebraic theory was developed in [1]. These notions are used in order to study the spectrum of a closed linear relation A in a Banach space in terms of the ascent, descent, nullity and defect of the relation $A - \lambda$, where λ is a complex number. Certain classes of linear relations are characterized.

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Stability for perturbed vector valued problems

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We present some necessary and sufficient conditions for the approximative stability of a perturbed vector valued problem.

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Normed coercivity for monotone functionals

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A normed coercivity result is established for order nonsmooth functionals fulfilling Palais-Smale conditions. The core of this approach is an asymptotic type statement obtained via local versions of the monotone variational principle in Turinici [An. St. UAIC Iasi, 36 (1990), 329-352].

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Section 5
Analytical and numerical methods in mechanics

Finite-dimensional Lie algebroids. Applications to classical mechanics

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In the last ten years the geometry of Lie algebroids was intensively studied and applied especially in Mechanics and Systems Theory. In this lecture we review our contributions in this field and add several new ones. We start with the vector bundles endowed with an anchor to tangent bundle of its base and we enlarge this notion to that of Lie algebroid. Then we introduce and derive various properties of the notion of connection in a Lie algebroid. Next, Riemann metrics on Lie algebroids are considered and a type of Levi-Civita connection is found. The geodesics of it are characterized as integral curves of a special semispray. Finsler structures on Lie algebroids are also discussed. The notion of semispray is treated in the framework of anchored vector bundles as the largest class of vector bundles for which such a notion can be considered. For a Lie algebroid we prove that any Lagrangian on it induces a semispray. Finally, we define mechanical systems with external forces in a Lie algebroid.

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Quasi-Hadamard product of some uniformly analytic and p-valent functions

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In this paper we study the quasi-Hadamard product between some pvalent and uniformly analytic functions with negative coefficients defined in connection with starlikeness and convexity.

Monochromatic plane waves in the linear theory of micropolar thermoelasticity

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The dynamical linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects (see Eringen [1] and Nowacki [2]). One of the most monograph investigations devoted to general and particular problems of elastostatics, elastodynamics, and thermoelasticity for the Cosserat medium is the book of Nowacki [3].

The basic equations of dynamic linear theory of micropolar thermoelasticity of an isotropic, homogeneous, and centrosymmetric elastic solid occupying a region $B \subset \mathbb{R}^3$, in terms of displacement vector $\mathbf{u}$, rotation vector $\boldsymbol{\varphi}$, and increment of temperature $\theta = T - T_0$ in $B \times \mathbf{T}_+$, where $\mathbf{T}_+$ is a time interval, in the absence of body forces, body couples and heat sources, are

$$\begin{cases} \Box_2 \mathbf{u} + (\lambda+\mu-\alpha)\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\mathbf{u} + 2\alpha\boldsymbol{\nabla}\times\boldsymbol{\varphi} - \nu\boldsymbol{\nabla}\theta = \mathbf{0}, \\ 2\alpha\boldsymbol{\nabla}\times\mathbf{u} + \Box_4\boldsymbol{\varphi} + (\beta+\gamma-\varepsilon)\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{\varphi} = \mathbf{0}, \\ D\theta - \eta\boldsymbol{\nabla}\cdot\mathbf{u} = 0, \end{cases} \tag{7}$$

where $\boldsymbol{\nabla}$ is the Hamilton operator, ∇^2 is the Laplace operator, and

$$\Box_2 = (\mu+\alpha)\nabla^2 - \rho\partial_t^2, \quad \Box_4 = (\gamma+\varepsilon)\nabla^2 - J\partial_t^2 - 4\alpha, \quad D = \nabla^2 - \frac{1}{\kappa}\partial_t.$$

The case of monochromatic plane waves of this theory is investigated.

A monochromatic plane wave is represented in the form

$$\mathbf{u} = A\mathbf{a}\,e^{[i(\zeta\mathbf{x}\cdot\mathbf{p}-\omega\,t)]}, \quad \boldsymbol{\varphi} = B\mathbf{b}\,e^{[i(\zeta\mathbf{x}\cdot\mathbf{p}-\omega\,t)]}, \quad \theta = C\,e^{[i(\zeta\mathbf{x}\cdot\mathbf{p}-\omega\,t)]}, \tag{8}$$

where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{p}$ are unit vectors, and A, B, C are constants. The wave is propagated in the direction of vector $\mathbf{p}$, while $\mathbf{a}$ and $\mathbf{b}$ define the directions of displacement vector $\mathbf{u}$ and rotation vector $\boldsymbol{\varphi}$, respectively.

Introduction of (8) into system (7) yields

$$\begin{cases} \{\rho\omega^2-(\mu+\alpha)\zeta^2\}A\mathbf{a}-(\lambda+\mu-\alpha)\zeta^2A(\mathbf{a}\cdot\mathbf{p})\mathbf{p}+2\alpha i\zeta B\mathbf{p}\times\mathbf{b}-i\nu\zeta\mathbf{p}C=\mathbf{0}, \\ \{J\omega^2-4\alpha-(\gamma+\varepsilon)\zeta^2\}B\mathbf{b}-(\beta+\gamma-\varepsilon)\zeta^2B(\mathbf{b}\cdot\mathbf{p})\mathbf{p}+2\alpha i\zeta A\mathbf{p}\times\mathbf{a}=\mathbf{0}, \\ -\omega\eta k\zeta A(\mathbf{a}\cdot\mathbf{p}) + (i\omega c_\varepsilon - k\zeta^2)C = 0. \end{cases}$$

The longitudinal displacement monochromatic wave (when $\mathbf{p} = \mathbf{a}$ and $\mathbf{b} = \mathbf{0}$), transverse waves (when $\mathbf{a}\cdot\mathbf{p} = 0, \quad \mathbf{b}\cdot\mathbf{p} = 0, \quad \mathbf{a}\cdot\mathbf{b} = 0.$), and longitudinal microrotational waves (case $\mathbf{p} = \mathbf{b}$ and $\mathbf{a} = \mathbf{0}$), are studied.

We find that neither the longitudinal microrotational wave nor the transverse waves are perturbed by the thermal waves. The only wave coupled with the temperature field is the longitudinal wave.

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The elasticity of the lattice structures with complex geometry

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The asymptotic behavior of the linear elasticity problem for two-dimensional lattice structures made by very thin and oblique bars is studied. These structures depend on the periodicity and the small thickness of the material. Our contribution consists in the homogenization of linear elasticity problem with respect to the thickness.

Improving preconditioned AOR iterative method for L-matrices

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In recent years, a number of preconditioners have been applied to solving of L-matrices with AOR method.

In this paper we use $(I + S'_\alpha)$ instead of $(I + S_\alpha)$ and $(I + S'_{\alpha\beta})$ instead of $(I + S_{\alpha\beta})$ comprised with Y.Li's precondition [1] and H.Wang's precondition [2] and obtain better convergence rate. Some numerical examples are also given that show that our preconditioner has better convergence rate.

Research of the is intense-deformed condition of elastic - plastic shells under the inuence of intensive dynamic loadings

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The problem of explosive loading of shell constructions (storages) is considered in this paper. Investigation of such problems is of considerable interest to reduce accidental risk and monitoring of the storages with flammable, explosive, or toxic fluids. Physical processes of loading and deformation are complex and of unstable character. Theoretical investigating stress-strain state and constructing mathematical model of the coupled problem leads to the necessity to take into account elastic-plastic behavior of materials, the process of formation and propagation of shock and depression waves and other factors.

A mathematical model was elaborated and basic relations for the coupled problem of soil and elastic shell interaction under loading are presented. Computer modeling of the loading dynamics is described in terms of state equation in the form of Mi-Gruneisen [1], taking into account complex stress-strain state of the matter. The variety of soil types with different mechanical-and-physical properties is presented as porous ternary medium (solid particles, water and air). A soils characteristics depends on volume content of each component, that under real conditions can vary over a wide range [3].

A difference scheme of second order of accuracy that is used for the numerical solution of the problem. This scheme is a developing of the Wilkin's scheme [1,2]. Computational experiments include the development of the expanding detonation products, shell loading and other physical processes. Qualitative physical picture of different phases is presented in two-dimensional plots which show the stress-strain state for different times.

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Fuzzy Technique in efficient solving of the Multiple Criteria Transportation Problem of special type

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In this paper we investigate the multi-objective transportation model of fractional type, assuming that each objective function has a fuzzy goal. Additionally we consider the time minimizing criterion. We developed an efficient solving procedure for this kind of problem. Applying the proposed method we obtain an efficient solution which is close to the best lower bound of each objective function minimizing the worst upper bound for the fixed time period. Iteratively we can find the set of all efficient solutions for entire time period.

Neuro-fuzzy adaptive control synthesis for autonomous flight control of an unmanned air vehicle

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In this paper a unitary adaptive output feedback control based on Kalman synthesis, neural network for compensation of feedback linearization errors and antisaturation fuzzy logic is presented. The application is developed to stabilize and control a small unmanned aerial vehicle (UAV).

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Section 6
Mathematics in Physics, Chemistry and Industry

Simulation air curtain flow in a vertical open display cabinet

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Computational Fluid Dynamics (CFD) tool is use at the various applications design in the food industry such as refrigeration, cold display and storage, ventilation, drying, sterilization and mixing. The purpose of this paper is the simulation air curtain flow in a vertical open display cabinet with four shelves by using CFD. Temperature homogeneity in food refrigeration systems is directly governed by the airflow patterns in the system. CFD simulation of airflow provides an opportunity to develop improved understanding of the underlying phenomena influencing system performance, which can lead to reduced temperature heterogeneity and increased effectiveness and efficiency of refrigeration systems. Air curtains are widely used in refrigerated display cabinet. The main purpose of the air curtain is to reduce the air exchange and hence heat and moisture transfer between the conditioned environment and the surrounding ambient. The air curtain is taken as a turbulent fluid and described by the conventional $k - \varepsilon$ turbulence model. The air curtain velocity profile and the temperature variation in various cross sections have been introduced as contour condition by using UDF (User Defined Function). CFD simulations have been realized for 3D domain in unsteady state based on experimental results.

On the Rudin-Shapiro wavelet packets

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For some applications, it is more convenient to have orthonormal bases with high frequency localization. Because the orthonormal wavelet bases in $L^2(R)$, have a frequency localization which is proportional to 2^j at the resolution j. So if j is large then the wavelet bases have poor frequency localization. This will be provided by the wavelet packets. In this paper we consider the well known Rudin- Shapiro polynomials as a class of wavelets to construct a sequence of wavelet packets.

Queuing system type $[SM|M|1|\infty]^N$ with semi-Markov flow in the average scheme

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We study asymptotic average scheme for semi-Markov queuing system by a random approach and using compensating operator of the corresponding extended Markov process. The average algorithm is established for the queuing process described of the number of claims in every node and by using the random evolution approach on the Banach space C3(R). The main tool to this end is the compensating operator of the extended Markov renewal process. The specific our queuing system is that series scheme is considered with phase merging procedure.

Application of the orthogonal polynomials in solving Hammerstein integral equations

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In this paper, the numerical solutions to nonlinear integral equations of Hammerstein type

$$y(t) = f(t) + \int_0^1 k(s,t)g(s,y(s))ds, 0 \leq t \leq 1$$

are investigated, where Chebyshev polynomials are applied to approximate a solution for an unknown function in the Hammerstein integral equation. A degenerate kernel scheme basing on Chebyshev polynomial combined with a collocation method is present. The rate of approximation solution converging to the exact solution is given. Finally we also give some numerical examples for efficiency of the given method.

Computational approach of the phaseportrait for the vortic flow model

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Computational fluid dynamics (CFD) becomes recently more and more mature. Its software tools are increasing in importance, and constitute a great challenge for the research in this area. A modern area where CFD has large applications is the mixing theory. With its mathematical methods and techniques, the mixing theory developed the significant relation between turbulence and chaos. The turbulence is an important feature of dynamic systems with few freedom degrees, the so-called "far from equilibrium systems", which are widespread between the models of excitable media. In the previous works, the study of the 3D non-periodic models exhibited a quite complicated behavior. In agreement with experiments, they involved some significant events - the so-called "rare events". The variation of parameters had a great influence on the length and surface deformations. The study of 2D - periodic and non-periodic- models revealed also useful statistical data for the behavior analysis. In order to unify the mixing theory, another flow model is approached, namely the vortic (unsteady) incompressible flow model. A special computational standpoint of its behavior using the MAPLE11 tools offers interesting data for further analysis.

Probabilistic characterization of the dynamical stochastic systems with final critical state

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In this paper the dynamical stochastic systems with final critical state are studied. For these systems the set of states is finite and the transition time of the system from one state to another is considered unitary. At every discrete moment of time the distribution of the states is known. The evolution of the system is finishing when the system remain for m consecutive moments of time in his given critical state, where m is a fixed natural number. Such systems are a particular case of the dynamical systems with final sequence states. For these systems are defined some descriptive random variables: the evolution time of the system, the first alert moments and the total alert time. In this scientific research a new numerical approach for determining the main probabilistic characteristics of these descriptive random variables are proposed and grounded. The computational complexity of the elaborated algorithms is polynomial.

Cubic spline method and fractional steps scheme to approximate the phase-field system with non-homogeneous Cauchy-Neumann boundary conditions

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A *scheme of fractional steps type*, associated to the nonlinear phase-field transition system with non-homogeneous Cauchy-Neumann boundary conditions, is considered in the present paper. To approximate the solution of the linear parabolic system introduced by such approximating scheme, a cubic spline method have been used. A stability result for this new approach is proved and some numerical experiments, like simulation of *separation zone* between the phases of the material that is involved in the solidificationprocess, are performed too.

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Finite volume method for electrostatic field calculation

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The problem of determination of three-dimensional distribution of electric field potential in multiply-connected domain is considered [1]. We consider sufficiently omnibus technique for elaboration of some numerical models for electromagnetic fields in nonhomogeneous structures. The technique is based on block discretization and finite volume method ideas [2]. The a priori and the a posteriori analysis of the discrete solutions accuracy for bodies with complicated geometry demonstrates that such an approach offers some advantages in comparison with finite difference or with finite element methods. The elaborated algorithm is implemented for particular problem and some numerical results are given.

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Stability of weighted approximate schemes

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The mathematical modeling of the problem to transfer of any substance in atmosphere depend of three main factors diffusion process, absorbtion of substance and advection-convection process. The classical model of this evolution problem with one space variable x has the form

$$\frac{\partial \varphi}{\partial t} + v(x)\frac{\partial \varphi}{\partial x} - d(x)\frac{\partial^2 \varphi}{\partial x^2} = f(x,t), \qquad (1)$$

where $\varphi(x,t)$ is an unknown function and $f(x,t)$ is a source term (cf.[1]). Different numerical methods for approximate solving of this problem have been proposed: finite difference method, explicit and implicit Euler method, Crank-Nicholson method etc. In order to study the main properties of these numerical methods (such as the error estimation, the convergence of approximate solutions and the stability), the initial equation must be written in general form

$$\frac{\partial \varphi}{\partial t} + A\varphi = f,$$

where the linear operator $A : X \to Y$, and X, Y are infinite dimensional normalized spaces. Applying any numerical methods this problem can be reduced to following equation

$$D^n \varphi_i^n + A_i^n \varphi_i^n = f_i^n,$$

defined in finite dimensional normalized spaces $X_n \quad and \quad Y_n$. This equation represents the approximate scheme of the initial problem.The spectral properties of approximate operator in analysis of the stability of this scheme is studied in [2]. In recent years many authors use the fractional space derivative for modeling the process described by equation (1)(cf.[3]). In this paper the following equation is considered

$$\begin{aligned}
&\frac{\partial \varphi}{\partial t} - d_+(x)\frac{\partial^\alpha \varphi}{\partial_+ x^\alpha} - d_-(x)\frac{\partial^\alpha \varphi}{\partial_- x^\alpha} = f(x,t),\\
&\varphi(x,0) = s(x), \qquad (2)\\
&\varphi(L,t) = 0, \qquad \varphi(R,t) = b(x),
\end{aligned}$$

where $1 < \alpha \leq 2$.

Numerical investigations of stability of the coupled system pilot-airplane using the methods of frequency domain analysis

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In this paper the authors present a synthesis of their current work regarding a man-machine interaction: the pilot-airplane couple resulting in pilot induced oscillations. Naturally, this leads to a stability problem of the closed loop system (pilot-machine) that can be tackled by the methods of frequency domain analysis. The theoretical airplane used is the delta-canard Admire model.

On the Ramo's Theorem applied to semiconductor photodetection

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Nuclear or optical radiation detection requires temporal formation and integration of an electrical current pulse to deliver an electrical charge pulse containing incident radiation energy information. Analysis of electrical charge carriers (electrons and holes for semiconductor) leads to Ramo's theorem [1] which allow to build current pulse temporal shape. The paper presents the mathematical framework and a generalized analytical treatment of Ramo's theorem to form the current pulse for collection of a certain distribution of mobile electrical charges generated in the general configuration of collection electrodes. A first application is made for the case of a mobile electrical charges distribution generated by the exponential absorption in a semiconductor. The importance of the role of transit time and direction of propagation of optical radiation, taking into account different mobility values for electrons and holes, is highlighted.
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Adsorption models for treatment of experimental data on remove fluorine from water by oxihydroxides of aluminum

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The study was made of the adsorption of fluorine ions from aqueous solutions by oxihydroxides of aluminum. The equilibrium sorption was explained using the Lengmuir, Freundlich, Bet and Redlich-Peterson models of isotherms. The results obtained allow one to conclude that the mathematical model of adsorption Freundlich is the best for describing the measured experimental data, which testifies to the heterogeneity of the surface. The parameters of all equations of adsorption were calculated.

Section 7
Mathematics in Biology and Environmental Sciences

Some properties of integro-differential equations from Biology

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We present some models of integro-differential equations from population dynamics, where the integral term describes the nonlocal consumption of resources. Fredholm property of the corresponding linear operators are useful to prove the existence of travelling wave solutions. For some models, this can be done only when the support of the integral is sufficiently small. In this case, the integro-differential operator is close to the differential one. One uses a perturbation method which combines the Fredholm property of the linearized operators and the implicit function theorem. For some other models, Leray-Schauder method can be applied. This implies the construction of a topological degree for the corresponding operators and the establishment of a priori estimates for the solution. Some biological interpretations follow from this study.

Direct Global Optimization by Cellular Exclusion in a Bioconcentration Problem

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Models of practical interest used to determine a set of unknown parameters lead to numerical global optimization problems. Since most of the known computational methods conduct to local instead of global extrema or require an expensive numerical effort, constructing efficient algorithms for global optimization remains of current interest. It is well known that cellular exclusion is a powerful tool for locating the solutions of strongly nonlinear algebraic systems of equations. In this work, we show how cellular exclusion can be successfully adapted for *direct* solving global optimization problems. Here, we use this technique to determine unknown parameters associated to a bioaccumulation model governed by a system of ODEs. Finally, we analyze and compare the results, advantages and disadvantages provided by the direct and indirect cellular exclusion methods in our parameter estimating problem.

The intrinsic transmission dynamics of tuberculosis epidemics in the conditions of Moldova Republic

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Let us consider the system of three autonomous differential equations of type (1) which model the intrinsic transmission dynamics of tuberculosis epidemics (see [1], [2])

$$\frac{dS}{dt} = P - \lambda S - \mu S, \quad \frac{dL}{dt} = (1-p)\lambda S - \delta L - \mu L,$$

$$\frac{dT}{dt} = \delta L + p\lambda S - (\mu + \mu_T)T, \quad \lambda(t) = \beta T(t). \tag{1}$$

Parameters and variables of the system (1) are described in the table.

Parameters	Description
$S(t)$	number of susceptible individuals at time t
$L(t)$	number of infected individuals at time t
$T(t)$	number of infectious individuals at time t
$\lambda(t)$	force of infection per-capita at time t
Π	influx of young individuals
μ	natural death rate
p	probability of fast progression of diseases
δ	constant of reinfection progression rate of TB infection
μ_T	TB mortality rate
β	transmission parameter of TB infection

We constructed the operators of the widest finite-dimensional Lie algebra L_5, admitted by system (1)(see [3],[4]). Also system (1) was examined for statistical data[5] of Moldova Republic for 2005-2009 period. It was established that the intrinsic transmission dynamics of tuberculosis epidemics leads to decrease the number of infected and susceptible people and to increase the number of infectious individuals. As a result fast population decrease has been observed.

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Optimal protocols applied to natural response of the immune system in obesity-related chronic inflammation

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Chronic inflammation within fat tissue is now recognized as a contributor to the many ill health consequences that come with obesity, from diabetes to cardiovascular disease. In this work, the optimal control theory is applied to an extended version of the model introduced by P. Díaz et al. in [1]. The model is defined by a system of ordinary differential equations and reflects the molecular and cellular interactions of the macrophages, T cells, chemokines, and cytokines that cause chronic inflammation. Seeking to maximize the effect of drug treatments to the model, we use a control representing the treatment. The optimal control is characterized in terms of the optimality system, which is solved numerically for several scenarios.

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Mathematical models of endotoxin tolerance

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The general problem to be considered is the construction of mathematical models for endotoxin tolerance. In some previous works, ([1],[2], [3]) the authors presented some mathematical models for endotoxin tolerance.In [4] a Kopelman type model for fractal kinetics is applied to preconditioning phenomenon in enzymatic reactions . In the present work the authors continue the study of reaction kinetics in fractal environements by using Savageau type models and applying them to endotoxin tolerance.

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Lyapunov functionals for abstract disease propagation models

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We discuss the dynamics of certain general disease propagation models encompassing a number of iterations of the SIS, SIR and SIRS models with nonlinear incidence and variable population size, as well as a HBV spreading model. To this purpose, we construct suitable Lyapunov functionals which yield global stability results under biologically reasonable sufficient conditions, the existence of a threshold parameter which controls the stability of the system being obtained in certain particular cases.

Statistical Analysis of the Variation of the Population of Predatory Mites in the Natural Forests of Bucegi

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Distribution and temporal variation of the population of the predatory mites (Acari: Mesostigmata, Gamasina) in three forests ecosystems, Abies alba, Pices abies and Fagus sylvatica, from Bucegi Massif are analyzed. The analysis focusses on the seasonal and forest effects of the assemblage of the mites population and the time association of the species.

A Non-Newtonian fluid model with resistance parameter of uniform and non-uniform portion of arterial stenosis

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This work studies the critical points of a model of early tumor growth when proliferating rate and killing rates of tumor cells are constant, proliferating rate is time dependent and killing rate is constant. It is found that critical point of tumor cells decreases when proliferation rate increases and critical point increases when killing rates are increasing for constants rates. Critical point also increases when killing rate increases for time dependent killing rate and constant proliferation rates. Further, we also show that tumor cells increases when time increases for time dependent proliferation rate and killing rate.

Improved methods to compute the boundary layer functions of the first order asymptotic approximation of the Fitz Hugh-Nagumo model

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Based on a double expansion of the vector field in the FitzHugh-Nagumo model, some improved methods to compute the functions z and y of the first order asymptotic approximation of the model are presented. A method based on series for the integration of the boundary layer functions is also developed.

Fuzzy inference systems for estimation Air Quality Index

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This work presents the use of fuzzy techniques for Air Quality Index (AQI) assessments. In Romania, AQI is established on a scale from 1 to 6 (1-excellent, 6-very bad) using data acquired by automated stations of the National Network of Air Quality Monitoring (RNMCA).

Section 8
Theoretical Computer Sciences

Using Process Graphs to Analyze BPMN 2.0 Models

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Business process modeling is an active area of multidisciplinary research. A modeling standard widely used in industry is the recently updated BPMN - Business Process Model and Notation, now at version 2.0. We introduce the notion of process graph for an executable subset of the language, BPMN_exe, as an equivalent formal representation. We study what properties process graphs should posses in order to be well formed and show how they can be used as a verification technique.

Queuing system type $[SM|M|1|\infty]^N$ with semi-Markov flow in the average scheme

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We study asymptotic average scheme for semi-Markov queuing system by a random approach and using compensating operator of the corresponding extended Markov process. The average algorithm is established for the queuing process described of the number of claims in every node and by using the random evolution approach on the Banach space C3(R). The main tool to this end is the compensating operator of the extended Markov renewal process. The specific our queuing system is that series scheme is considered with phase merging procedure.

Methodological aspects for realization of an informational system for a continuous training of public officials in Moldova

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Human resources in public administration are essential in the sustainable development and state building. Human resource development is a complex process which is addressed both at national and international levels. Nationally, this process is influenced by the government policy in continuous training. The inefficiency of this process, due to the lack of transparency in the selection of trainees and the information that does not cover their needs, transforms the occurring investments into expenditures. The improvement of this process consists in the creation of a national database regarding the assessment and continuous training of elected officials and other staff in public administration bodies. Given the current need for increased administrative capacity of public administrations bodies fully compliant with European values, transparency, predictability, responsibility, adaptability and efficiency could be defined as the objectives of the sustainable development strategy of human resources in PA in Moldova. The development of human resources information system of government might be a viable solution for the sustainable development of this domain. The informational system could serve as an available managerial tool at the disposal of central and local authorities to justify the decisions regarding human resources in PA activity.

A scalable genetic algorithm for constructing of identifying sequences for VLSI circuits

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Genetic algorithms (GA) are widely used for solving the problems for constructing of identifying sequences of different types for VLSI [1]. To sequences of this type include: test, initialization, synchronization, etc. In GA's of this type, each individual is an input binary sequence. To evaluate the fitness-function for individuals a fault or fault-free simulation algorithm of VLSI are used depending on the problems type. Due to this fact, these algorithms are relatively slow. If used multi-core workstation, parallel computation of evaluation functions on processor cores is a good way to increase the performance. In [2] we considered algorithms of this type for workstations with 2-4 cores. In this work we consider the problem of constructing of parallel genetic algorithm for multi-core workstation. In our approach, a function, that evaluates the fitness of a single individual, is realized as a thread class. Creating the necessary number of objects of this class, it is easy to adapt the algorithm for workstations with different numbers of processors (cores). To test the efficiency of the proposed approach, we chose an algorithm for constructing of initialization sequences [3] in which for evaluation the fitness of individuals is used fault-free simulation of VLSI. We carried out a series of computer experiments with ISCAS-89 benchmarks circuits on Intel's 12-core workstation. The achieved speed up of work-time of GA varies from 4 to 13.5 times, the average 8,09 times. Maximum speed up of work-time is achieved when the number of simultaneous threads varies from 12 to 128, an average of 80, which is considerably higher than the number of cores in the system.

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On Pareto-Nash equilibrium

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We investigate the notion of Pareto-Nash equilibrium as continuation of precedent works [2, 4]. The problems and the basic theoretical results are exposed. The method of intersection of graphs of best response mappings [3] is applied for solving the dyadic two-criterion games. This paper represents the results of Pareto-Nash instant games investigation. By applying the generalization to well known notions and by applying the synthesis function method and the method of intersection of best response graph, the conditions for the Pareto-Nash solutions existence are formulated and demonstrated. The method for determining the Pareto- Nash equilibrium set for the dyadic two-criterion games in mixed strategy is constructed [5]. Illustration examples are provided. A development of the method, with implication of the computer science technologies, is done.

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On Independent Sets in Unicyclic Graphs

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A set $S \subseteq V$ is independent in a graph $G = (V, E)$ if no two vertices of S are adjacent. The *independence* and *matching numbers* of G are denoted by $\alpha(G)$ and $\mu(G)$, respectively. If $\alpha(G) + \mu(G) = |V|$, then G is called a *König-Egerváry graph*. The number $d_c(G) = \max\{|A| - |N(A)| : A \subseteq V\}$ is called the *critical difference* of G, and $X \subseteq V$ is *critical* if $|X| - |N(X)| = d_c(G)$ [**?**]. By core(G) we mean the intersection of all maximum independent sets, while $\ker(G)$ denotes the intersection of all critical independent sets. The graph G is called *unicyclic* if it is connected and has a unique cycle, which we denote by $C = (V(C), E(C))$. Let $N_1(C) = \{v : v \in V - V(C), N(v) \cap V(C) \neq \emptyset\}$, and $T_x = (V_x, E_x)$ be the tree of $G - xy$ containing x, where $x \in N_1(C)$, $y \in V(C)$.

It is known that $\ker(G) \subseteq \text{core}(G)$ for every graph G [1], while $\ker(G) = \text{core}(G)$ for bipartite graphs [2]. Our main finding claims that if G is a unicyclic non-König-Egerváry graph, then $\ker(G) = \text{core}(G) = \cup\{\text{core}(T_x) : x \in N_1(C)\}$. It is worth mentioning that these equalities are satisfied by some, but not all, unicyclic König-Egerváry graphs.

Problem. *Characterize unicyclic König-Egerváry graphs with the following property:*

$$\ker(G) = \text{core}(G) = \cup\{\text{core}(T_x) : x \in N_1(C)\}.$$

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Independence polinomials of the unicyclic graphs whose Fibonacci indexes are extremal

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An independent set in a graph G is a set of pairwise non-adjacent vertices, and the *independence number* $\alpha(G)$ is the cardinality of a maximum stable set in G. The *independence polynomial* of the graph G is $I(G;x) = s_0 + s_1x + ... + s_{\alpha(G)}x^{\alpha(G)}$, where s_k equals the number of independent sets in G of size K (I. Gutman and F. Harary, [2]).

The *Fibonacci index* $Fib(G)$ (or *Merrifield-Simmons index*) of G is the number of all its independent sets, i.e.,$Fib(G) = I(G;1)$, [3]. Tight lower and upper bounds for Fibonacci indexes are known for general graphs, connected or not, unicyclic or not, and the corresponding extremal graphs are characterized [1], [3], [5]. Recall that the graph G is called *unicyclic* if it is connected and has a unique cycle. In this note we give explicit formulae for independence polynomials of these extremal unicyclic graphs, which we furher use to find again their Fibonacci indexes.

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Comparison between Parameters of Anti-Collision Algorithms For Active RFID

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RFID is one of the most popular technologies for identification, which will have a huge development in the future. Many Active RFID systems have more advantages in comparison with passive RFID systems. The main problem raised in this study is represented by the collision problem. Many algorithms are and have been proposed, but few of them can be trusted. We will analyze different comparisons between parameters of anti-collision algorithms and present improved variants vulnerabilities of ALOHA and DCMA algorithms.

Numerical algorithms regarding Polling systems with exhaustive service

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The Polling systems find wide applications in modeling and design of transport and industrial processes, cellular networks and broadband wireless networks [1]. We'll present and discuss new numerical algorithms for solving important characteristics appeared in analysis of exhaustive Polling systems [2], such as Kendall and Pollaczek-Khintchin equations, probabilities of state, etc. Numerical algorithms are elaborated using programming languages C++ and C #.

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Some generalizations of Kendall and Pollaczek - Khintchin equations for Polling systems

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The queueing systems of Polling type play an important role in analysis and management of broadband wireless networks [1]. One of the important characteristics of these systems is the k - busy period [2]. In [3] is showed that analytical results for k - busy period can be viewed as generalization of classical Kendall functional equation [4]. In this work will be presented and discussed the no stationary queue length distribution for exhaustive Polling models with semi - Markov switching. Will be shown that this distribution can be viewed as an analog of famous Pollaczek - Khintchin transform equation [5, 6]. Some numerical algorithms will be discussed.

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Singly connected graph clustering

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In this paper a new algorithm for graph clustering will be presented. Based on the property of 'singly-connectedness', the algorithm takes an iterative approach in order to merge various nodes. In the end, the resulting graph will have the property of singly-connectedness. The algorithm focuses on directed graphs, although similar algorithms can be constructed for undirected graphs. The majority of the existing clustering algorithms are based on the notion of 'distance'. While this approach yields good results, there are some particular cases in which the distance is irrelevant, or could be misleading. The singly-connected graph clustering does not rely on the notion of 'distance', instead it only checks the connectedness of nodes. Moreover, the resulting graph will respect the singly-connected property. The algorithm has applications in theoretical mathematics, theoretical and applied computer science, as well as any field where the singly-connected property is of relevance. In particular, the singly-connected property is specific to a high level software pattern - the software layer - and it can be used in order to reverse-engineer an application's source code in an attempt to identify its software layers.

Using State Machines for modeling and testing software systems

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The specification of software systems with Finite State Machines models is a well known and widely used technique. One of the main applications of these models is the usage in comparing the software systems runtime behavior with the expected behavior defined by the specification. By trying to determinate

if the match exists, several runtime scenarios are practiced witch are known as test cases. When a test case identifies an unspecified runtime behavior means that the software system contains a faulty implementation. One of the potential problems is the possibility of unauthorized access to the data processed by the software system. A possible solution to avoid the memory exposure due to potential faulty implementation is to refine the design and to minimize the access of the software system to its own memory. The article proposes this design and discusses his advantages.

Kernel Independent Component Analysis in edge detection

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Kernel methods are successfully applied in signal processing. This paper proposes a method for applying kernel independent component analysis in the field of edge detection. The proposed algorithm partitions the test image in neighborhoods of size m and uses each of them as input for the Kernel ICA processor. The result is then analyzed in order to extract masks of size m that are further used for edge detection. Our experimental results prove the higher success rate as compared to standard edge detection methods.

On BufferZone Automata Net - related case study -

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DDoS type attacks represent the most frequent attacks on the internet. For now, no viable method to stop these has been found, this issue generating cost of millions of dollars for their prevention. In this article we provide a functionable solution to terminate these attacks. The solution is based upon the model of another security issue: the validation of a message sent from an unknown source. Thus, we shall create a BufferZone Automata Net, which will be responsible for the autheticity of every recieved message. The BufferZone Automata Net will be a secuirty interface between the browser and the server. The propose of this paper is to compare our solution with an agent based solution, one of the best solutions on the market. We shall demonstrate that BufferZone Automata Net is far better than the other one, because it stops the attacks not only "try" to prevent them.

On advanced databases with uncertain information

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Advanced databases allow to store and manage large volumes of data, coping with the rapidly increasing amount of information needed in most of the research, medical, economic or industrial fields. On the other side, the specific data that some domains might provide can be uncertain or may contain errors. In such databases, the problem is still both to provide a good management, the possibility to retrieve the requested information and to appropriately support the decision making, even if one works with uncertain data. In this context, the concept of random database has become important. The paper will present the formalization of the relational operations and some results of this domain. Also, some particular cases, in which databases contain columns of different probability distributions, will be described. (This work was supported by the strategic grant POSDRU/89/1.5/S/58852, Project "Postdoctoral programme for training scientific researchers" cofinanced by the European Social Found within the Sectorial Operational Program Human Resources Development 2007 - 2013).

Using Petri nets as a formalism to specify distributed systems

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Petri nets are a mathematical model used for the specification and the analysis of parallel and distributed processes. They were introduced by C. A. Petri in the early 1960s as a graphical and mathematical tool for modelling such processes, and they proved to be suitable formalism for describing and studying information processing systems that are characterized as being concurrent, asynchronous, distributed, parallel, nondeterministic, and/or stochastic. As a graphical tool, Petri nets can be used as a visual-communication aid similar to flow charts, block diagrams, and networks. As a mathematical tool, it is possible to set up state equations, algebraic equations, and other mathematical models governing the behaviour of systems. Petri nets are a powerful language for system modelling and validation. They are now in widespread use for a very wide variety of applications because of their generality and adaptability. They have been successfully used for concurrent and parallel systems modelling and analysis, communication protocols, performance evaluation and fault-tolerant systems. Due to their numerous applications in areas like engineering, economics, medicine, education and science, Petri nets became a veryprolific research field soon after their introduction, such that today the Petri nets research is materialized in a large number of publications in prestigious journals and conference proceedings.

On the serial connection of the regular asynchronous systems

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The asynchronous systems f are multi-valued functions, representing the non-deterministic models of the asynchronous circuits from the digital electrical engineering. In real time, they map an 'admissible input' function $u : \mathbf{R} \to \{0,1\}^m$ to a set $f(u)$ of 'possible states' $x \in f(u)$, where $x : \mathbf{R} \to \{0,1\}^n$. When f is defined by making use of a 'generator function' $\Phi : \{0,1\}^n \times \{0,1\}^m \to \{0,1\}^n$, the system is called regular. The usual definition of the serial connection of systems as composition of multi-valued functions does not bring the regular systems into regular systems, thus the first issue in this study is to modify in an acceptable manner the definition of the serial connection in a way that matches regularity. This intention was expressed for the first time, without proving the regularity of the serial connection of systems, in our work *Some properties of the regular asynchronous systems*, published in the International Journal of Computers, Communications and Control, Vol. 3, 2008, pp 11-16. Our present purpose is to restate with certain corrections and prove Theorem 45 from that work.

Section 9
Education

On the limit of a function in a point

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The material was prepared in a demonstration lessons, in partnership with Technical College "Baros Gabor" in Debrecen (Hungary). I tried to present the type of lesson plan and outline the type of existing lesson Romanian education system. Lesson was held in class XII-E route progressive, where five students are integrated with disabilities, students who have completed Sat exam, and are without capacity. Tries to present material the basic ideas of the lesson "Limits of functions in a given point" to be accessible to all categories of existing students in the classroom.

Applying Mathematical Models through the Prism of Integrating Factor in the Study of Biological Process

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In the process of study of biological processes a significant role belongs to the development and examining of various math models. Thus, mathematical modeling may achieve connections among different biological compartments, contributing in a very special way to the research of phenomena under review. In such a way, naturally via the developed mathematical models, the integration of biological qualitative and quantitative knowledge is performed. So, what does model and modeling stand for? The Model is the system that does not differ from the real object, the examined phenomenon, in relation to some properties, considered essential and differs by other properties, considered to be non-essential. Concurrently, the lack of non-essential elements in the built model is not less significant than the presence of essential elements. The process of developing and applying the built model is called modeling. The mathematical model will stand for system of mathematical relations that describe the essential properties of the phenomenon or the process under study. The working mathematical models are reduced to building via the known mathematical apparatus of an abstract simplified similitude of the created system or process. There are several emphasized parameters and further, changing their value, depending on the goal, the behavior of the given system shall be researched. From our standpoint, three important categories of biological problems can be underlined, which can be approached in the university level program for biology via the mathematical models: Category 1. Problems that are associated with the identification of various characteristics of biological systems. This category of problems can be solved through differential equations. Example of problems: 1. Which are the optimal conditions under which the maximal stable number of a population of organisms can be? 2. Which are the mechanisms and laws of adapting of organisms (with warm blood) to the changes of environment (e.g. change in the temperature of air)? Category 2. The problems which study the totality of homogeneous biological objects. In order to solve the problems from this category the probabilistic and statistical models are applied. Examples of problems: 1. To determine how many phenotypes exist, having exactly n-dominant characters? 2. How many types of gametes will the heterozygote hybrid provide after n-independent characters? Category 3. Problems that study the inter-dependence among two or more characteristics of the examined biological phenomenon. The given problems can be solved applying the mathematical models that deal with approximation of functions. Example of problems: 1. To determine the correlation expressed through an analytical function between the duration of vegetative phases and the vegetative on the one hand, and the length of floral axis on the other hand? 2. To identify the analytical function that expresses the interdependence between the dynamics of basipetal opening of the inflorescence and the duration of flowering phase?

Thus, the same mathematical models can represents the description of various biological processes which at first glance may seem very different and no connection may be noticed among them. This is how the mathematical modeling may represent the integrating factor of various compartments from biology.

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Using logical inference scheme to solve geometry problems

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The article presents in the first part the role in the formation of logical inference schemes resolution strategies and how to use the problem-solving strategies. The second part of the paper contains how to use a practical method in logical inference schemes for solving geometry problems. The conclusion highlights the importance of this category of cognitive strategies in solving geometry problems.

Problem Posing in the Upper Grades Using Computers

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The problem posing activity of students in mathematics classroom has been observed to have rich outcomes in profound understanding of mathematics and in fostering the problem solving ability or creative disposition. We describe practical ways of problem posing using computer, based on our previous studies on problem posing activities for university students (prospective teachers) and high school students. Studies on activities of problem posing have been scarce for high school and university level students, henceforth upper grade students, and classroom practices of the activities are less known. We first identify essential points of problem posing in the upper grades using computers, and introduce practical activities. We report surveys on some of our concrete activities of problem posing and demonstrate their validity. We also present the effects of computer use in problem posing in our setting.

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On the application of informational technologies in the teaching process of "Modeling and numeric computation" from the high-school informatics course

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In the teaching-learning process of the lessons of "Modeling and numeric computation", an important role is kept by the informational technologies. In this work we examine the ability of the pupils to elaborate mathematical models, to compose algorithms for the numerical methods and we discuss their capacity to analyze the obtained results.

Regarding the didactical transposition competences of the mathematical contents

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The article addresses the initial professional formation problem of the future teachers of mathematics. In particular, the professional skills of mathematical contents didactical transposition necessary essential to the teachers in the formal and non-formal educational process are examined.

The role of graphical representation in making the study of mathematics in gymnasium more efficient

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For the gymnasium pupils, the graphical representations may be challenging in learning mathematics because they bring an increase in the understanding of the given material. The illustrations and the different types of representations may help in the understanding of the text, the graphical organizers and representations, the images, facilitate the understanding of the scientific concepts.

The consequent and adequate usage of the graphical representations by the mathematical teacher, in every type of activities (teaching, learning, consolidation, evaluation), as well as the habit of pupils of realizing tables, diagrams, maps, when they have to compare, verify, quantify or structure the information, simplify the assimilation of information process, creates the motivation of learning. The formal activities must be completed by activities of non-formal education, of application of theory, of information, research and investigation.

The use of rectangle "pavement" and coloring to solve some problems

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The paper refers to the use of "pavement" and coloring the rectangle to solve problems. Solving these types of problems contribute to the development of the imagination, logical thinking, and intuition. The selection of the problems related to the "pavement" allowed us to generalize some methods and simplify the ways of solving them.

The idea of the article emerged while solving the following problem proposed at the 26th edition of the Cities' Tournament held in Russia, on the 17th of October 2004:

Let A and B be rectangles. It is known that using rectangles congruent to the rectangle A, one can pave a rectangle homothetic to rectangle B. Prove that one can pave a rectangle homothetic to the rectangle A using rectangles congruent to rectangle B.

Students' difficulties in composing and decomposing problems

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The problem analyzed in this paper requires decompositions or compositions of numbers or figures. In a broad sense, composing means reconstitution of the whole from known parts. The parts (components) are well identified, we have the properties of the whole and we want to (re)build it. Decomposing means breaking down a whole into parts. The properties of the parts are well defined in the problem, but their identification can be problematic. We analyze students' answers in composing and decomposing problems, proposed to a multiple choice contest. We found that this type of problems have a high rate of correct answers, with some exceptions. The first exception was observed for optimization problems. These problems require successive approximations, a particularity that seems to explain the lower success rates for them. A second exception was observed for "pure" and "mixed" problems for which students record significant differences. Our data suggest that shifting between domains offers multiple ways for verification and, therefore, success rate is higher for mixed problems than for the domain specific ones.

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Bilingual students in school environment: the case of George

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In Romania, there are a growing number of students for whom Romanian language is not their first language. These students experience many challenges adjusting to new educational environments. Therefore, teachers need support in understanding and responding to the linguistic and cultural challenges that these students face in learning mathematics. In the paper, we present such a situation, and some effective solutions to overcome it. We analyze here the case of George: he is a Chinese 5th grader. George is not fluent in Romanian language; nevertheless, he studies now in an ordinary middle school in Romania. For George it is a real challenge to understand text word problems, even more than to effectively solve these problems. We present the methods the first author used to help George to overcome the difficulties he had in communicating in Romanian, and in understanding word problems. We found that involving George in problem posing activities could be a real opportunity for him to better understand the text of a problem.

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Using Origami to teach Math concepts

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A lot of studies (as for example Pope, 2002) pointed out that Origami could be used as an instructional tool in education. In this paper we present an empirical experiment concerning the use of Origami activities in learning geometrical concepts. These activities were conducted during a summer camp for students in grades 4 to 6. After folding an Origami model, students was asked to answer some math questions concerning the given model. We looked therefore to Origami as a method to develop student's mathematical knowledge. We found that, during these activities, students improved their ability to use a mathematical language to explain geometrical properties. Therefore, Origami could be a beneficial method of math instruction.

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On some crucial points in the process of teaching - learning mathematics

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We discuss the idea of crucial points, points of change, in the learning process, i.e. moments of time in which a progress is attained, an essential jump is performed, and after which the learning process is realized by the help of the ideas and possibilities of the respective change. We bring some older and some newer examples.

In what regards the numerical computation, a crucial point is represented by the decimal writing of numbers, that relies on ideas coming from the Babylonian, Chinese or Indian antiquity, but only in Renaissance it was introduced in Europe (not without the resistance of many persons) [1], pag. 326.

Even if the mathematical symbolism is present along the centuries and became present everywhere in XVIth century, in what regards the symbolic calculus, the crucial change is due exclusively to François Viète (1540-1603) that creates the mathematical language, a prototype of the programming languages [2], in which the subsequent mathematics is exposed.

A major change in worldwide culture is given by the modern electronic technique. We mention the softwares MAPLE, MATLAB, Mathematica and so on. Because of the computers, the study of mathematics has the chance of becoming easier, more pleasant, more efficient - precious pedagogical and psychological qualities. Some effort of adaptation is required, some resistance is opposed, the values of the past are invoked. Also many problems appear: the inertia of the teaching staff, due to some minuses; the lack of teaching materials done in accordance with the pedagogical and psychological requirements; the reconsideration of curricula by the introduction of laboratory classes, the acquisition of the necessary computers for the lessons and so on.

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DNA Computing - overview -

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Leonard Adleman's experiment from 1994 showed that machines that can perform computation relying on DNA. That experiment is a remarcable achievement since, for the first time, humankind could compete with the genius involved in the bio-chemical mechanisms.

Reflexive assembly via DNA Computing

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This paper concerns the concept of reflexive assembly. It is structured in three parts.

In the first part I will present the notions of DNA computing, Tiling theory and DNA Nanotechnology. In the second part I will expose the field of self assembly. I will illustrate how it works by a counting program. Finally, I will present a discussion of the pluses and minuses of reflexive assembly.

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